Centrality-based aggregate ratings in two-sided markets — with an application to on-line Dating

Daniele Condorelli*

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Preliminary and incomplete

1 Motivation

Many markets are two-sided. In two-sided markets, participants can be typically partitioned in two groups, in a such a way that interaction take place exclusively (or almost) between members of different groups. The dating market, populated by men and women, is a traditional example. Car-sharing markets where a neat separation between the role of drivers and passengers can be drawn are another example.

Often, two-sided markets are managed by firms, also called "platforms", who operate upstream to both categories of users. Platforms, such as dating websites or car-sharing companies, reduce search costs by allowing users to find each other easily and reduce transaction costs by formalizing the rules under which the two sides interact.

In pursuing their objectives, platforms will often want to make market outcomes contingent on some ranking of their users. For instance, a dating website may provide advantages to users who are deemed more attractive, and a car-sharing platform may punish badly rated drivers. Typically, the ranking will be build based on some observations of the users interactions or by asking users to rate each other. To continue with the previous examples, the dating website may be observing how many times a certain user has been selected as a potential match (e.g., see Tinder), while the car sharing platforms may ask their users to explicitly rate their interaction with the other party on a numerical scale (e.g., see Uber).

*Department of Economics, University of Essex, Colchester, UK. E-mail: d.condorelli@gmail.com.

Sometimes, platforms may wish to develop a ranking that assigns more weight to judgments of more highly-rated people. For example, it is conceivable that, in seeking to determine an overall attractiveness ranking of men and women, a dating application may want to give more weight to the judgment of more attractive users. Similarly, car-sharing platforms may wish to downplay bad reviews of drivers by passengers who have themselves a lot of bad reviews. Analogously, a bad review by a bad behaved driver could be given less weight than the review made by a virtuous one.

In this note, I put forward a system for ranking users in two-sided markets which accomplishes this goal. The system I propose determines a score based on eigenvector centrality, which is a widely adopted measure of influence in a graph. In particular, the score of a user on one side of the market is determined as a weighted average of the ratings received by users on the other side, where weights are determined by the scores of said users. And vice-versa.

Eigenvector centrality has been applied to ranking problems in one-sided markets. Palacio-Huerta and Volij (2004) use eigenvector centrality to obtain a ranking of academic journals based on the network of cross-citations. Page et al. (1998) describe a method for ranking web-pages based on the hyper-link network structure, which then formed the basis for the Google algorithm. In these two works, the network does not have a bipartite structure. An application of eigenvector centrality to bipartite graphs is discussed in Daugulis (2012). While the mathematics underlying the computation of eigenvector centrality is closely related, the author does not consider the case of independent ratings by the two sides, which is what is required by the applications I consider.

To illustrate the main ideas in this note, in the next section I develop the proposed method using as an example the problem of obtaining an attractiveness ranking of users in dating applications. The environment discussed is, intentionally, maintained as simple as possible. Further analysis is required to extend the ideas to more complex environments.

2 Attractiveness ranking in dating platforms

Suppose there is a finite set of men, $\mathcal{M}$, and women, $\mathcal{W}$, participating to a dating platform. Next, assume that each woman has rated every man and, vice-versa, each man has rated every woman. Let $M = (m_{i,j})$ be a matrix that represents the evaluation of women towards men. That is, for each man i and woman j, matrix element $m_{i,j}$ contains the rating that $j \in \mathcal{W}$ assigns to man $i \in \mathcal{M}$. Equivalently, let $W = (w_{j,i})$ represent the matrix of ratings of men toward women. We assume these matrices are both positive, hence $m_{i,j} > 0$ and $w_{j,i} > 0$ for each $j \in \mathcal{W}$ and $i \in \mathcal{M}$.

As a first step, we normalize these two matrices. Let $m_i = \sum_i m_{i,j}$ and $w_i = \sum_j w_{j,i}$ be the sum of the ratings given by man i and woman j, respectively. Then, let $D_M = diag(m_i)_{i \in \mathcal{M}}$ and

$D_W = diag(w_i)_{i \in \mathcal{W}}$ indicate diagonal matrices where the main diagonal contains the sum of the ratings provided by each man and woman. We let $\overline{M} = MD_M^{-1}$ and $\overline{M} = WD_W^{-1}$ represent the normalized matrix of ratings. It is immediate to see that both $\overline{M}$ and $\overline{W}$ are stochastic matrices since, for both of them, the elements of each column (i.e., the ratings of any given user) sum to one.

We are looking for a numerical valuation for each man and for each woman such that the valuation of any man (woman) is the weighted average of the normalized ratings assigned to him by women (men), were weights are determined according to the valuations assigned to women (men). We let m represent the vector of valuations assigned to men and w the vector of valuations assigned to women. We are looking for positive m and w that solve the following system:

$$m = \overline{M}w$$
$$w = \overline{W}m.$$

Now, note that by substituting the unknown variable on the right-hand side of both equations, the above can be rewritten as

$$m = \overline{MW}m$$
$$w = \overline{WM}w$$

and observe that both $\overline{MW}$ and $\overline{WM}$ are column-stochastic as they are the product of two column stochastic matrices. Moreover, they are irreducible, as every element is strictly greater than zero.

We can then use Perron-Froebnius theorem to find solutions, m^* and w^* to the above equations. As it is well known, the eigenvectors m^* and w^* corresponding to the 1-eigenvalues of the two matrices $\overline{MW}$ and $\overline{WM}$ solve the two equations above. Note that the theorem also implies that both m^* and w^* are unique up to scaling.

Concluding, the vectors m^* and w^* correspond to our attractiveness indexes for men and woman. In particular, the higher the index, the more attractive is the dating application user according to our measure.

I now provide an example to illustrate the potential usefulness of the approach. Let's assume there are 3 women and 4 men and users rank each other from most attractive to least attractive. In particular, a men will attribute a rating of 3 to the most preferred woman, a rating of 2 to the second best one and a rating of 1 to the one he likes least. Let M and W be as follows:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 1 & 1 \\ 3 & 3 & 3 & 3 \end{pmatrix} \quad M = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{pmatrix}.$$

In words, the first column of W suggests that the first man prefers woman 3 to woman 2 to woman 1. This ranking is shared by man 2. Men 3 and 4 also consider women 3 as the most attractive, but they prefer woman 1 to woman 2. Instead, M indicates that all women share the same preferences, whereby man 4 is preferred to man 3 who is preferred to man 2, who is preferred to man 1.

If we were to rank users based on the average rating, woman 1 and woman 2 would be tied in the second position, as both have the same average rating of 3/2, while woman 3 would take the top position with an average rating of 3. However, when we compute our attractiveness index (normalized to 1) we obtain that $w^* = (0.28, 0.22, 0.5)$. That is, woman 1 is now considered more attractive than woman 2. This is so because she is preferred by the two men who are highest ranked, given $m^* = (0.1, 0.2, 0.3, 0.4)$.

3 References

Daugulis, P. (2012), "A note on a generalization of eigenvector centrality for bipartite graphs and applications", Networks

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