# Commitment via Third-Party Contracts in Bilateral Trade: A Three-Way Equivalence*

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**Abstract**

A buyer with private value makes a take-it-or-leave-it offer to a seller with private cost. Which trading outcomes are implementable if the seller can sign observable and binding contracts ex ante with a third party that specify transfers as a function of the price posted and whether trade occurs? We establish a *three-way equivalence*: contract-implementable outcomes coincide with those achievable if the seller commits ex ante to an observable cost-dependent price-acceptance strategy, which are outcomes implementable with direct bilateral trading mechanisms subject to ex post monotonicity of the allocation in the seller's cost, and buyer's interim incentive and participation constraints. An upstream firm that charges royalties to a seller facing a monopsonist buyer can implement ex post efficiency, but also turn a buyer monopsony into a seller monopoly.

**Keywords:** contracts as commitment devices, monopoly pricing, buyer power, mechanism design, vertical contracts

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# 1 Introduction

Contracts with third parties are a recognised source of strategic commitment (Schelling, 1960). Yet, little is known about how they expand the set of feasible outcomes in canonical trading environments with informational frictions. We analyse ex ante vertical contracts in a model of bilateral trade with two-sided private information. Before a price-posting buyer makes an offer, a seller signs an observable, binding contract with a third party. We fully characterise the downstream outcomes these contracts can induce, showing that for the seller they are are as effective as committing to an acceptance strategy. At one end of the Pareto frontier, royalty schemes can turn buyer monopsony into seller monopoly; at the other, they can restore ex post efficiency, allocating all surplus to the buyer.

Consider a private-value buyer purchasing a good from a seller with privately known costs. The ex ante optimal mechanism for the buyer is monopsony pricing, i.e., making a take-it-or-leave-it offer. Suppose that, before types are realised and prices posted, the seller signs a public, non-renegotiable contract with the third party (e.g., the lincensor of a technology, or a platform providing market access). This contract specifies a monetary transfer contingent on the observable terms of trade, specifically the price offered by the buyer and whether the transaction occurs. Our goal is to characterise all *contract-implementable outcomes*: pairs of functions from types to allocation and buyer-to-seller payment that can arise in the buyer's monopoly pricing game under some agreed ex ante contract.

We prove a three-way equivalence (Theorem 1). A trade-contingent royalty schedule specifying price-dependent payments with a sale-independent fixed fee — to which we show we can restrict attention (Lemma 2) — can contract-implement any equilibrium outcome that arises when the seller commits ex ante to an observable, monotone, cost-based acceptance strategy, before types are realised and the price is posted. In turn, this is the set of outcomes implementable by a direct bilateral-trading mechanism, as in Myerson and Satterthwaite (1983), with payments in posted-price form (i.e., the ex post transfer is proportional to the ex post allocation, with a proportionality factor that depends only on the buyer's value and uniquely pins down the resulting transfer given cost), such that the buyer's interim incentive compatibility and participation constraints hold, and the allocation rule is ex post non-increasing in the seller's cost.

We leverage Theorem 1 to characterise the buyer-seller ex ante Pareto frontier within contract-implementable outcomes, assuming ex ante that the budget between the seller and the third party is balanced and that neither sustain a loss (Theorem 2). At one extreme, there is a contract that neutralises the buyer's market power, allowing the seller to obtain an ex ante profit equal to that achievable by posting prices. Thus, a royalty scheme can turn a monopsony of the buyer into a monopoly of the seller. At the other, the buyer-preferred contract induces ex post efficient trade and yields the first-best surplus to the buyer, leaving the seller and third party at their reservation payoffs. The Myerson and Satterthwaite (1983) impossibility result is sidestepped as the contract replaces the seller's interim participation constraint with an ex ante one.

These two results are intuitive if asymmetric information is one-sided (Corollary 2.2). Suppose, for instance, that the seller's cost is public. Efficient trade is feasible without a contract because the buyer offers to pay the known cost when their value is above it. Conversely, the seller becomes a monopolist by signing a resale-price-maintenance agreement which imposes prohibitive liquidated damages should the seller accept any price lower than the monopoly price associated with that cost. Facing such contractual commitment by the seller, the buyer offers the monopoly price whenever their value is higher.

However, consider the seller-optimal outcome when both cost and valuation are private. Then, different types have different ideal prices. For each cost $c$, the optimal posted price for a monopolist seller is equal to $\psi^{-1}(c)$, where $\psi$ is the buyer's virtual valuation (Myerson, 1981). Hence, because $c$ is unknown ex ante and unobserved by the buyer, a contract that forces all of the buyer's types to offer a unique price is suboptimal.

Instead, the seller-optimal contract-implementable outcome is achieved via a royalty scheme that induces the buyer to self-select into the desired pricing strategy. For any monotone buyer's pricing strategy, $p_{\mathcal{B}}(v)$, define royalty payments

$$k(p) = p - \psi(p_{\mathcal{B}}^{-1}(p)).$$

This royalty schedule ensures that the seller accepts exactly when its cost $c$ is below the virtual valuation of the buyer type who would offer $p$, i.e., when $c \leq p - k(p) = \psi(p_{\mathcal{B}}^{-1}(p))$. Thus, $k$ implements the seller-optimal allocation of the object.

We prove the result by establishing the existence of a monotone and incentive compatible $p_{\mathcal{B}}$. Then, the Payoff Equivalence Theorem (Myerson, 1981) implies the equilibrium offer of a buyer is the expected payment conditional on trade that they expect to make in the seller's monopoly mechanism. Equivalently, it is the equilibrium bid of a single bidder in a first-price auction with random reserve price, $p_{\mathcal{B}}(\psi^{-1}(c))$.

The problem above can also be viewed through the lens of non-linear pricing (Mussa and Rosen, 1978). For given $k$, the buyer's choice of price $p$ selects a trade probability $q = G(p - k(p))$, where $G$ is the CDF of cost. Implementing a higher $q$ requires procuring from progressively higher-cost sellers, which generates a convex cost. Thus, royalties act as a screening instrument that reshapes the buyer's effective quality–price menu. As a consequence, the seller-optimal contract can be interpreted as solving a Mussa–Rosen-type problem in the one-dimensional allocation variable $q$.

The royalty contract that implements the seller-optimal outcome normally involves both positive and negative payments. Higher cost seller are subsidised to accept prices they would not otherwise, and lower cost ones are deterred from accepting too low prices. However, we show on average the royalty paid is zero. Consequently, there is no need to use a fixed fee for the third-party to break-even. Although, a profit-driven third party who had full bargaining power against the seller, for example a licensor of an essential technology, would choose a fixed fee to fully extract the monopoly profit.

Conversely, the buyer-optimal outcome is contract-implemented through a royalty scheme which usually involves only negative payments. In this case, the optimal royalty implements an ex post efficient outcome. Because the seller observes the price before making a choice, their decision must be optimal ex post. It follows from Green and Laffont (1977) that the seller must be made a residual claimant over the entire surplus. To do so, the royalty is set at

$$k(p) = p - p_{\mathcal{B}}^{-1}(p),$$

given a buyer's monotone pricing strategy $p_{\mathcal{B}}(v)$. As a result, the seller accepts any offer $p_{\mathcal{B}}(v)$ whenever $c \leq p_{\mathcal{B}}^{-1}(p_{\mathcal{B}}(v)) = v$. Because $p_{\mathcal{B}}(v) \leq v$, the royalty paid is always negative and must be recouped by a breaking-even third-party via a fixed fee that complements the royalty schedule. Meanwhile, the seller also breaks even, because the buyer offers a price equal to the expected cost of production conditional on a sale,

$$p_{\mathcal{B}}(v) = \mathbb{E}_c[c \mid c < v].$$

Normalised by the probability that the offer is accepted, this payment is the expected externality the buyer imposes on the seller. Thus, it is the payment the buyer would make in the Expected Externality Mechanism of d'Aspremont and Gérard-Varet (1979) and, therefore, is incentive compatible given the resulting allocation is ex post efficient.

Clearly, allowing unrestricted third-party contracting increases total welfare if the buyer-optimal contract is signed. However, turning a monopoly of the buyer into one of the seller has, in general, ambiguous effects on total welfare. Hence, we do not provide a clearcut welfare argument in favour or against the countervailing use of ex ante royalty contracts. Nonetheless, we are able to show that if the seller's monopoly generates higher welfare than the buyer's monopsony, the total welfare effect of allowing ex ante contracting is positive no matter which point in the frontier is chosen (Proposition 1). Seller-optimal royalties are the welfare worst case among Pareto optimal contracts.

**Implications** Since they do not play any other role in our model, we isolate a novel rationale for royalty schemes as a commitment device to countervail the power of a strong buyer. The applied conclusion of our analysis is that such contracts are extremely powerful in shaping downstream behaviour. In industrial organisation, a large literature examines two-part tariffs as means to leverage market power from a monopolised upstream market to a competitive one downstream (e.g., see Rey and Tirole (2007)). We demonstrate monopolisation is possible even in the presence of buying power and despite informational frictions between the upstream firm and the seller. Thus, we mitigate claims that buyer power will temper anticompetitive behaviour by an upstream monopolist in the supply chain (e.g., see Dobson and Waterson (1997)).[1]

We show that negative royalties are an essential feature of Pareto optimal contracts, one antitrust authorities may want to scrutinise further when evaluating vertical arrangements. While we are not aware of a documented case where they have been used to countervail market power, negative royalties are often in place. For example, pharmaceutical companies often contract with distributors or licence to other manufacturers, who then go on selling to large pharmacy chains, national purchasing agencies or consortia of hospitals. Such distribution or licensing agreements may include price-maintenance clauses with large liquidated damages and involve rebates that turn royalties from payments into subsidies for larger sales.

A contract that gives all surplus to the buyer won't be entered willingly into by a seller and a self-interested third party. Hence, it is less likely to be of practical relevance. However, it is conceivable an efficiency-seeking government agency that is unable to regulate the downstream pricing of a seller may wish to license it in a way that further

[1]Arguably, the strong buyer might attempt to sign a contract first, initiating a commitment race. While the observation has merit in theory, we take the view that contracts between elements of a supply chain are more credible and observable than those signed with outside brokers.

raises surplus in a final market, even if the market is already competitive because of buying power. As a possible example, observe that U.S. offshore oil and gas leasing combines a high upfront bonus bid for market access with ongoing royalties that can be partially suspended or reduced through statutory royalty-relief programs. Interpreted through our lens, the bonus bid plays the role of the fixed fee, while the relief functions like an output-contingent rebate that encourages higher production even when the downstream operator has substantial bargaining power.

To bring our insights to bear on the antitrust of vertical contracts, it is useful to emphasise load bearing assumptions. We devote the concluding Section 5 of the paper to this. However, let us mention here two crucial assumptions: contract observability and non-renegotiability. Analogously to the argument made by Hart and Tirole (1990) and O'Brien and Shaffer (1992) for the case of multiple price-posting sellers, if the contract is secret or can be renegotiated after the buyer has posted the price, the third party and the seller would agree on the ex post optimal rule of accepting the buyer's offer if above cost. Thus, any unobserved contract must be assumed to impose no royalties.

Notwithstanding, firms and governments have many opportunities to make the contracts they signed public and hard to renegotiate. Contracts are often announced to the press and details disclosed in public filings. A prominent example is Apple's 2010 agency agreements with major book publishers in the US, whose public terms were designed to credibly shift pricing power away from Amazon with its \$9.99 per book policy and formed the core of the ensuing antitrust litigation. The shift to the agency model was announced in January 2010 alongside the launch of the iBookstore. Major news outlets (The New York Times, Wall Street Journal) explicitly reported on the key terms: publishers would set retail prices (between \$12.99 and \$14.99 for new releases), and Apple would take a 30% commission.

**Related Literature** A large body of work explores the insight that contracts allow commitment.[2] Our setting — with observable, binding contracts preceding an interaction with asymmetric information — bridges two strands of this literature.

The first is optimal delegation, where a principal grants public decision-making authority to an agent. While early work focused on oligopoly (Vickers, 1985; Fershtman, Judd, and Kalai, 1991), recent literature endows the agent with private information and emphasises balancing flexibility against control (Alonso and Matouschek, 2008; Armstrong and Vickers, 2010). Closest to our paper is Thereze and Udayan (2025), who characterise the set of outcomes implementable through delegation. In a model of bilateral trade, they show that ex post efficient trade is often unattainable when the principal is restricted to allocating decision rights. We build on this framework by embedding delegation in a richer contractual environment. The third-party contract governs, via royalties, the degree of discretion a privately informed seller has when negotiating with a buyer. By allowing side payments in addition to the allocation of authority, our contracts overcome the impossibility in Thereze and Udayan (2025) and restore full efficiency.

The second strand of literature is contracting with externalities, most notably Aghion

[2]Some recent work has focused on commitment in itself: Bade, Haeringer, and Renou (2009) and Bizzotto, Hinnosaar, and Vigier (2022).

and Bolton (1987) and Segal (1999). Aghion and Bolton show how a seller with known cost and buyer can sign an exclusivity contract to extract rent from a third-party entrant with private cost. We analyse the structural inverse: a seller and a third party signing a contract to extract rent from the buyer. The presence of two-sided asymmetric information flips the welfare implications: while Aghion and Bolton find that exclusivity reduces total surplus, we show that seller's commitment can improve efficiency. Segal shows that a principal contracting individually with multiple agents can obtain an ex-post efficient outcome in the presence of multilateral externalities if the contract of each agent can condition on all messages. We also show ex-post efficiency is attainable, but in contrast to Segal the principal in our framework may only write contracts with one of the two agents.

We align with Loertscher and Marx (2022) in interpreting the canonical mechanism-design approach to trade as one in which the buyer possesses substantial bargaining power. Extending this perspective to bilateral trade with two-sided asymmetric information, we show that observable, non-renegotiable vertical contracts with a third party can countervail buyer power by operating as an ex ante commitment device. While Loertscher and Marx (2022) and much of the subsequent work emphasise the implications of horizontal mergers, our results imply vertical integration can be counterproductive because it eliminates the commitment value inherent in arm's-length contracting.

Other related work on buyer power and vertical restraints analyses how contractual arrangements affect negotiated outcomes through outside options and exclusion incentives (e.g., Ho and Lee, 2019; Chambolle and Molina, 2023). We complement these approaches by providing a complete characterisation of the outcomes implementable via third-party royalty schedules, and by mapping the ex ante Pareto frontier under asymmetric information, thereby delineating the scope for contracting to countervail buyer power without restricting attention to particular contractual forms.

We provide a mechanism-design characterisation of observable, non-renegotiable third-party vertical contracting in monopoly pricing. Theorem 1 can be read as a type of revelation principle: the set of outcomes implementable via royalty schedules coincides with the set implementable by a direct bilateral-trade mechanism under specific constraints. A closely related insight — arriving via information design rather than side contracting — is in Ichihashi and Smolin (2023), where a buyer-side recommendation rule acts as a commitment device and the induced payoff frontier can be characterised using standard screening tools.

Our results also clarify why ex post efficiency becomes attainable once the seller's interim participation constraint is effectively replaced by an ex ante one: the third party plays the role of a budget breaker, a classic device for restoring efficiency when bilateral budget-balance or renegotiation constraints would otherwise bind (e.g., see d'Aspremont and Gérard-Varet (1979) and Baliga and Sjöström (2009)). Related dynamic environments in which early contracting relaxes participation and incentive constraints include Giovannoni and Hinnosaar (2024). Finally, from a more technical viewpoint, the structure of our ex ante Pareto frontier is closely aligned with the geometry of one-dimensional screening and non-linear pricing in regulation and monopoly pricing (Baron and Myerson, 1982; Mussa and Rosen, 1978).

# 2 Model

A seller, $\mathcal{S}$, and a buyer, $\mathcal{B}$, bargain over the sale of a single object. The buyer makes a take-it-or-leave-it price offer, $p \in \mathbb{R}$, to $\mathcal{S}$. The value of the object to $\mathcal{B}$, denoted $v$, and the marginal cost of production of $\mathcal{S}$, denoted $c$, are private information of $\mathcal{B}$ and $\mathcal{S}$. We assume $v$ and $c$ are drawn from CDFs $F$ and $G$ with $\text{Supp}(F) \subseteq [\underline{v}, \overline{v}]$ and $\text{Supp}(G) \subseteq [\underline{c}, \overline{c}]$, respectively. We will say that $F$ (or $G$) is *regular* if it admits a density $f$ (or $g$) which is everywhere positive on $[\underline{v}, \overline{v}]$ ($[\underline{c}, \overline{c}]$) and the hazard rate $\frac{f}{1-F}$ (or reciprocal reverse hazard rate $\frac{G}{g}$) is increasing.

We wish to characterise the set of outcomes that can arise in the monopoly pricing game above when, prior to learning its cost and bargaining, $\mathcal{S}$ has signed an observable and irrevocable contract with a third party. The contract specifies what $\mathcal{S}$ pays to the third party as a function of the actions taken during the bargaining game and possibly the realisation of some public random variable, $\omega \sim U[0, 1]$. Define any such contract as a measurable and uniformly bounded function $[0, 1] \times \{0, 1\} \times \mathbb{R} \ni (\omega, x, p) \mapsto m(\omega, x, p) \in \mathbb{R}$ where $x \in \{0, 1\}$ indicates whether trade has taken place ($x = 1$) or not ($x = 0$) and $p \in \mathbb{R}$ is the price offered by $\mathcal{B}$.

Assuming $m$ is in force and the state is $(\omega, v, c)$, ex post payoffs for the buyer and the seller are:

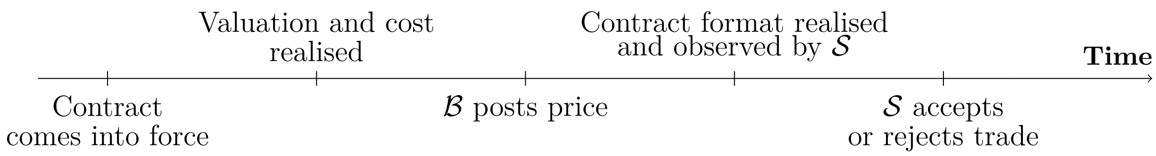
$$u(x, p\,; v) := x(v - p)$$
$$\pi(x, p, m\,; c, \omega) := x(p - c) - m(\omega, x, p).$$

An important role will be played by the profit of the seller gross of the payment to the third party, which we will refer to as $\mathcal{S}$'s *trade surplus*:

$$\hat{\pi}(x, p\,; c, \omega) := \pi(x, p, m\,; c, \omega) + m(\omega, x, p) = x(p - c).$$

Figure 1 illustrates the timing of the model.

**Figure 1:** Timeline




We assume the uncertainty of the contract is resolved after $\mathcal{B}$ posts its price, but before $\mathcal{S}$ decides whether to accept trade or not. Making the contract a random function of the price expands implementation possibilities in a useful way, but only as long as uncertainty is resolved prior to acceptance. However, our key results specialise to the case of a deterministic contracts which we discuss following Theorem 1.

Given a contract is in force, a strategy for $\mathcal{S}$ is price-acceptance mapping $[0, 1] \times [\underline{c}, \overline{c}] \times \mathbb{R} \ni (\omega, c, p) \mapsto a(\omega, c, p) \in [0, 1]$, while $\mathcal{B}$'s strategy is a price mapping $[\underline{v}, \overline{v}] \ni v \mapsto p(v) \in \mathbb{R}$. A (Perfect Bayesian) equilibrium under contract $m$ is a profile of strategies $(p, a)$ such that play is sequentially rational. Let us denote with $\mathcal{E}(m)$ the set of equilibria of the bargaining game arising given contract $m$.

## 3 A Three-Way Equivalence

An *outcome* in the bilateral trading game is a pair of functions

$$[\underline{c}, \overline{c}] \times [\underline{v}, \overline{v}] \ni (c, v) \mapsto (q(c, v), t(c, v)) \in [0, 1] \times \mathbb{R}$$

which specifies for each profile of types the probability the object is traded and the expected payment made from the buyer to the seller. Following Myerson and Satterthwaite (1983), each outcome corresponds with a *direct mechanism*, whereby seller and buyer simultaneously report their types and the mechanism designer implements $(q, t)$. A profile of strategies $(p, a)$ *induces* outcome $(q, t)$ if

$$q(c, v) = \mathbb{E}_\omega[a(\omega, c, p(v))]; \quad (1)$$
$$t(c, v) = p(v) \cdot \mathbb{E}_\omega[a(\omega, c, p(v))]. \quad (2)$$

We look for outcomes that arise as equilibrium for some contract between the third-party and the seller. The following definition introduces the outcomes of interest.

**Definition 1** *An outcome $(q, t)$ is* contract-implementable *if there exists contract $m$ and equilibrium $(p, a) \in \mathcal{E}(m)$ that induces $(q, t)$.*

We shall say that $m$ contract-implements $(q, t)$ with (ex ante) *budget balance* if there exists $(p, a) \in \mathcal{E}(m)$ that induces $(q, t)$ with no ex ante transfer of funds between the seller and the third party, that is

$$\mathbb{E}_{\omega,v,c,x\sim Bern(a(\omega,c,p(v)))}[m(\omega, x, p(v))] = 0.$$ [3]

Since we assumed no budget constraints and because if $(q, t)$ is contract-implemented by $m'$, then it is also implemented by $m' + z$ for any fixed real valued $z$, we conclude that any contract-implementable outcome can be implemented with budget balance.

The next Lemma identifies the set *contract-implementable* outcomes. Outcomes associated with direct mechanisms in posted-price form that are incentive compatible and interim individually rational for the buyer, and whose allocation rules exhibit ex post monotonicity in cost can be contract implemented, and no more.

**Lemma 1** *Outcome $(q, t)$ is contract-implementable if and only if:*

1. *For each $v$, $q(c, v)$ is non-increasing in $c$;*
2. *$\int_{\underline{c}}^{\overline{c}} q(c, v)dG$ is non-decreasing in $v \in [\underline{v}, \overline{v}]$;*
3. *$\int_{\underline{c}}^{\overline{c}} t(c, v)dG = \int_{\underline{c}}^{\overline{c}} \left[vq(c, v) - \underline{v}q(c, \underline{v}) + t(c, \underline{v}) - \int_{\underline{v}}^{v} q(c, x)dx\right] dG$;*
4. *$\int_{\underline{c}}^{\overline{c}} [q(c, \underline{v})\underline{v} - t(c, \underline{v})]\, dG \geq 0$;*
5. *$t(c, v) = q(c, v)d(v)$ for some $d : [\underline{v}, \overline{v}] \to \mathbb{R}$ where $(q, t)$ factors through $d$.*[4]

[3]A random variable $X$ has distribution $Bern(s)$, parametrised by $s \in [0, 1]$, if $X$ takes values in $\{0, 1\}$ and $X = 1$ with probability $s$.

[4]Outcome $(q, t)$ *factors through* $d$ if for each $c$, $v \mapsto (q(c, v), t(c, v))$ is constant over the fibres of $d$: for any $v, v' \in [\underline{v}, \overline{v}]$, $d(v) = d(v') \implies (q(c, v), t(c, v)) = (q(c, v'), t(c, v'))$ for all $c \in [\underline{c}, \overline{c}]$. Given $t(c, v) = d(v)q(c, v)$, if $d(v) > 0$ for all $v$ then $t$ factoring through $d$ implies the outcome $(q, t)$ also factors.

Conditions 2–4 are classic and ensure interim incentive compatibility and individual rationality for the buyer.

Condition 5 ensures the transfers fit the posted price structure: if transfers satisfy this condition we shall say they are in *posted-price form*. Formally, the ex post transfer must be proportional to the ex post allocation. Moreover, the proportionality factor must depend only on the buyer's value and must pin down the resulting allocation and transfer rule for any given cost. If two buyers types post the same price in the monopoly pricing game, then the corresponding transfer in the mechanism must be the same and can depend on cost only through the allocation. The condition does not restrict the ex post allocation nor interim payoffs. For any allocation rule $q$ satisfying Conditions 1 and 2, there exists ex post transfers satisfying Conditions 3 and 5, simultaneously.[5]

Condition 1 requires monotonicity of the ex post allocation rule in cost. This condition is necessary for dominant strategy incentive compatibility on the seller side, which, once the third-party payment is taken into account, is forced by the timing of the monopoly pricing game. Moreover, it is sufficient for the existence of transfers $\tilde{t}$ such that $(q, \tilde{t})$ is dominant strategy incentive compatible for seller. However, in general, Lemma 1 does not imply $t = \tilde{t}$. Because the contract payments are ex-ante budget balanced but need not be ex-post, the seller's ex-post payoffs differ from those derived directly from $(q, t)$ and, instead, may be adjusted to coincide with those associated with $(q, \tilde{t})$. Specifically, to implement outcome $(q, t)$ contract payments are chosen equal to $t - \tilde{t}$ giving the seller an effective transfer of $\tilde{t}_{\mathcal{S}} := t - (t - \tilde{t}) = \tilde{t}$.

*Proof. Necessity:* Suppose $(q, t)$ is induced by $(p_m, a_m) \in \mathcal{E}(m)$ for some contract $m$.

Because $\mathcal{B}$ posts their price before $\mathcal{S}$ makes their acceptance decision, the effective transfer to seller, $\tilde{t}_{\mathcal{S}}$ — defined as the transfer from the buyer minus the contract payments — must be such that $(q, \tilde{t}_{\mathcal{S}})$ is dominant strategy incentive compatible for the seller. By Green and Laffont (1977), this implies $q(c, v)$ is non-increasing in $c \in [\underline{c}, \overline{c}]$ for each fixed $v \in [\underline{v}, \overline{v}]$, giving Condition 1.

Because $(p_m, a_m) \in \mathcal{E}(m)$ form an equilibrium and induce $(q, t)$, the outcome must be incentive compatible and individually rational for the buyer. Conditions 2, 3, and 4 then follow from Myerson (1981).

Finally by definition of $(p_m, a_m)$ inducing $(q, t)$,

$$
\begin{aligned}
t(c, v) &= \mathbb{E}_\omega[p_m(v) \cdot a_m(\omega, c, p_m(v))] \\
&= p_m(v)\mathbb{E}_\omega[a_m(\omega, c, p_m(v))] = p_m(v)q(c, v).
\end{aligned}
$$

Hence, we may take $d(v) = p_m(v)$. Since $q(c, v) = \mathbb{E}_\omega[a(\omega, c, p(v))]$ depends on $v$ solely through $p_m(v)$, $q(c, \cdot)$ is constant over the fibres of $p_m(v)$. Similarly, $t$ is also constant over $p_m$-fibres. Consequently, $(q, t)$ factors through $d = p_m$, giving Condition 5.

*Sufficiency:* Suppose $(q, t)$ satisfies the given conditions. We construct a contract $m$ with $(p_m, a_m) \in \mathcal{E}(m)$ inducing $(q, t)$.

[5] To simplify exposition, we assumed that the buyer never randomises over prices. All our analysis goes through if the buyer strategy is defined as $[\underline{v}, \overline{v}] \ni v \mapsto p(v) \in \Delta(\mathbb{R})$. In that case, Condition 5 is replaced by the requirement that transfers be in *mixed posted-price form*: $t(c, v) = \int t_i(c, v) d\tau = \int d_i(v) q_i(c, v) d\tau$ for a collection of outcomes $\{(q_i, t_i)\}_{i \in \mathcal{I}}$ with $\mathcal{I}$ an index set, $q_i(c, v)$ non-increasing in $c$ for each $v$, $\tau \in \Delta(\mathcal{I})$ such that $q = \int q_i d\tau$, for functions $d_i : [\underline{v}, \overline{v}] \to \mathbb{R}$ such that for all $i, j \in \mathcal{I}$ $d_i(v) = d_j(v') \implies q_i(c, v) = q_j(c, v'), \forall c$.

By Condition 2, the set of buyer types which trade with probability zero is (up to a null set) a possibly empty lower interval $[\underline{v}, \tilde{v})$ or $[\underline{v}, \tilde{v}]$ for some $\tilde{v} \in [\underline{v}, \overline{v}]$ — without loss, assume it is $[\underline{v}, \tilde{v})$. Define

$$p_m(v) := \begin{cases} v & \text{if } v < \tilde{v}, \\ d(v) & \text{if } v \geq \tilde{v} \end{cases}$$

We now construct contract payments. Let $m(\omega, 1, p) = \overline{v}$, deterministically for any $p < \tilde{v}$ — so as to dissuade $\mathcal{S}$ from trading with types $v < \tilde{v}$. Additionally, fix contract payments such that $m(\omega, 0, p) = 0$ for all $p \in \mathbb{R}$ and $\omega \in [0, 1]$.

By Condition 5, $d(v) = d(v') \implies q(c, v) = q(c, v')$ for all $c$. Therefore, we may define a function $\beta(c, p)$ with $\beta(c, p) = q(c, v)$ whenever $p = d(v)$ and $\beta(c, p) = 0$ if $p$ is not in the image of $d$. Since by Condition 1 $q(c, v)$ is non-increasing in cost for each $v$, we may consider $1 - q(\cdot, v)$ as a CDF over costs, possibly after adding a dummy cost value above $\overline{c}$ so the CDF reaches full probability mass. Let $H(c, p) = 1 - \beta(c, p)$. Then $\tilde{H}(\omega, p) := \inf\{c \in [\underline{c}, \overline{c}] \mid H(c, p) \geq 1 - \omega\}$ satisfies $\mathbb{P}_\omega(\tilde{H}(\omega, p) \geq c) = \beta(c, p)$.

Define $m(\omega, 1, p) = p - \tilde{H}(\omega, p)$ whenever $p \geq \tilde{v}$. The constructed contract is

$$m(\omega, x, p) := \begin{cases} x\overline{v} & \text{if } p < \tilde{v} \\ x(p - \tilde{H}(\omega, p)) & \text{if } p \geq \tilde{v} \end{cases}$$

By sequential rationality, seller accepts if and only if

$$p - c - m(\omega, 1, p) \geq -m(\omega, 0, p) \iff \tilde{H}(\omega, p) = p - m(\omega, 1, p) \geq c$$

Define the pure acceptance strategy

$$a_m(\omega, c, p) := \begin{cases} 0 & \text{if } p < \tilde{v} \\ \mathbb{1}\{\tilde{H}(\omega, p) \geq c\} & \text{if } p \geq \tilde{v} \end{cases}$$

By construction, $\mathbb{E}_\omega[a_m(\omega, c, p_m(v))] = \mathbb{P}_\omega(\tilde{H}(\omega, d(v)) \geq c) = \beta(c, d(v)) = q(c, v)$. Therefore,

$$\mathbb{E}_\omega[a_m(\omega, c, p_m(v))] = \mathbb{P}_\omega(\tilde{H}(\omega, p_m(v)) \geq c) = q(c, v)$$
$$\mathbb{E}_\omega[p_m(v) \cdot a_m(\omega, c, p_m(v))] = p_m(v)\mathbb{E}_\omega[a_m(\omega, c, p_m(v))] = d(v)q(c, v) = t(c, v)$$

Since $a_m$ is sequentially rational by construction, Conditions 2, 3, and 4 imply $(p_m, a_m) \in \mathcal{E}(m)$, so that $(p_m, a_m)$ induces $(q, t)$. □

Next, we show that the set of contract-implementable outcomes is essentially as large as the set of outcomes a seller can implement by committing to an interim price-acceptance strategy ex ante, i.e., before it learns cost.

To make this equivalence formal, let's envision a *commitment version* of our model, in which the contract with the third party is replaced by a (measurable) commitment strategy for $\mathcal{S}$, denoted $\alpha(c, p) \in [0, 1]$, specifying the probability with which type $c$ should accept price $p$. We say $\alpha$ is *monotone* if for $c > c'$, $\alpha(c, p) \leq \alpha(c', p)$. As with a contract, commitment strategies induce equilibrium outcomes. In an equilibrium, the buyer with value $v$ posts a price in $P(v,\alpha) := \arg\max_p \mathbb{E}_c[\alpha(c,p)](v-p)$ and the seller accepts according to $\alpha(c,p)$. Let $\mathcal{P}(\alpha) := \{p_\alpha : [\underline{v},\overline{v}] \to \mathbb{R} \mid \forall v \in [\underline{v},\overline{v}],\ p_\alpha(v) \in P(v,\alpha)\}$ be the collection of optimal price schedules given commitment strategy $\alpha$. We say commitment strategy $\alpha$ and $p_\alpha \in \mathcal{P}(\alpha)$ *induce* outcome $(q,t)$ if

$$
\begin{aligned}
q(c,v) &= \alpha(c,p_\alpha(v)) \\
t(c,v) &= p_\alpha(v)\alpha(c,p_\alpha(v))
\end{aligned}
$$

We now define outcomes implementable by a monotone commitment strategy.

**Definition 2** *Outcome $(q,t)$ is* commitment-implementable *if there exists monotone commitment strategy $\alpha$ and $p_\alpha \in \mathcal{P}(\alpha)$ that induces $(q,t)$.*

We are ready to state the main result of the paper. It establishes a mutual equivalence in terms of implementable outcomes between ex ante contracting of the seller with a third party, the seller's commitment to an interim acceptance strategy at the ex ante stage, and the power to design the allocation mechanism under constraints.

**Theorem 1** *The following statements are equivalent:*

*(i)* $(q,t)$ *is contract implementable;*

*(ii)* $(q,t)$ *is commitment implementable;*

*(iii)* $(q,t)$ *is a direct mechanism with ex post transfers in posted-price form and allocation monotone in cost, satisfying interim incentive compatibility and interim individual rationality for the buyer.*

A commitment strategy induces a contract-implementable outcome if and only if it is monotone. Hence, the set of outcomes implementable via commitment is strictly larger than the set of contract-implementable ones. However, monotonicity is a weak condition. If acceptance is non-monotone, then there exists a monotone strategy inducing the same distribution of prices and trade, hence generating the same revenue for the seller and buyer, yet weakly lower production cost.

Theorem 1 naturally extends to the case where the contract $m$ is required to be deterministic. In this case, Conditions 1 and 2 in Lemma 1 are replaced with $q(c,v) = \mathbb{1}\{\phi(v) \geq c\}$ for some non-decreasing $\phi$; a commitment strategy is defined as $\alpha(c,p) \in \{0,1\}$; and the direct mechanism described in (iii) above has an ex post allocation rule that takes a cut-off form.

*Proof.* Lemma 1 proved (i) $\iff$ (iii). That (i) $\implies$ (ii) is immediate by setting commitment strategy $\alpha(\cdot,\cdot) = \mathbb{E}_\omega[a(\omega,\cdot,\cdot)]$. We now prove (i) $\impliedby$ (ii).

Suppose $(q,t)$ is commitment implementable. Then there exists a monotone commitment strategy $\alpha$ and $p_\alpha \in \mathcal{P}(\alpha)$ inducing $(q,t)$. By definition, $q(c,v) = \alpha(c,p_\alpha(v))$ and $t(c,v) = p_\alpha(v)\alpha(c,p_\alpha(v))$. We show outcome $(q,t)$ satisfies the conditions of Lemma 1.

Because $q(c,v) = \alpha(c,p_\alpha(v))$, monotonicity of the commitment strategy implies Condition 1. Conditions 2, 3, and 4 follow from the fact $p_\alpha \in \mathcal{P}(\alpha)$ is optimally chosen by $\mathcal{B}$. Additionally, because $\mathcal{B}$ posts prices, the transfers must satisfy Condition 5.[6] □

[6] If buyer is permitted to randomise over prices, the proof is analogous given the mixed posted-price form definition given in Footnote 5.

In a contract (or commitment) implementable outcome, the seller may not receive enough money from the buyer to pay the cost of production. For example, a contract might commit the seller to accept any offer above the lowest possible cost. If we wish the seller to be paid enough to recover production costs then outcome $(q, t)$ must give a non-negative *ex ante trade-surplus* to the seller:

$$\hat{\Pi}(q,t) := \mathbb{E}_{v,c}[t(c,v) - q(c,v)c] = \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} t(c,v) - q(c,v)c \, dGdF \geq 0. \tag{S-IR}$$

Together with 1–5 from Lemma 1, (S-IR) is necessary and sufficient for an outcome to be contract-implementable with balanced budget and the seller not sustaining a loss, or as we shall say, to be contract-implementable *without production subsidy*. The condition is obviously necessary. If the outcome fails it, then either the seller or the third-party must be covering production cost in excess of revenues. It is sufficient because, as we argued, any outcome can be implemented by a budget-balanced contract and (S-IR) ensures that the earnings accrued from the buyer cover the cost of the seller when the net transfer to the third party is zero.

If we wish to specialise Theorem 1 to characterise contract-implementable outcomes without production subsidies in (i), then the conditions of Lemma 1 will have to include (S-IR) and the mechanism design formulation in (iii) will have to specify an ex ante participation constraint for the seller. Additionally, the definition of commitment-implementable in (ii) will need to restrict attention to monotone commitment strategies that are ex ante profitable.

Motivated by practical relevance, we close this section by showing it is without loss to restrict attention to contracts of the form $m(\omega, x, p) = xk(\omega, p) + T$, where $k$ is a *price-dependent royalty* paid only if trade occurs and whose realisation is observed by $\mathcal{S}$, and $T$ is a *fixed fee* paid irrespective of trade. Define this class of contracts as *two-part royalty*.

**Lemma 2** *For any $(q, t)$ contract-implementable by contract $m$ and $(p, a) \in \mathcal{E}(m)$, there exists two-part royalty contract $m'$ and equilibrium $(p', a') \in \mathcal{E}(m')$ inducing $(q, t)$ that generates the same expected transfer to the third party.*

Because $m(\omega, x, p) = xm(\omega, 1, p) + (1 - x)m(\omega, 0, p) = x(m(\omega, 1, p) - m(\omega, 0, p)) + m(\omega, 0, p)$, any contract can be rewritten as a two-part one, where a random price-dependent fee, $m(\omega, 0, p)$, is paid irrespective of whether trade occurs or not and another, $k(\omega, p) = m(\omega, 1, p) - m(\omega, 0, p)$, is only paid if trade takes place. The proof of Lemma 2 shows $m(\omega, 0, p)$ can be replaced with its expected value without affecting trading behaviour nor the interim payment made by $\mathcal{S}$ to the third party. The key is that a payment from $\mathcal{S}$ that is independent of whether trade takes place cannot influence the price acceptance decision of $\mathcal{S}$ nor, as a result, the price offered by $\mathcal{B}$.

## 4 Ex Ante Pareto Frontier

Having characterised contract-implementable outcomes, we now chart the ex ante buyer-seller Pareto frontier within it. We focus our search on outcomes that satisfy conditions 1–5 from Lemma 1 plus (S-IR). We insist on (S-IR) because in mapping the frontier it is natural to ask that any contract entered between the third-party and

$\mathcal{S}$ does not require an injection of external funds. We take trade-surplus, the sum of seller's profit and third-party's revenue, as the relevant metric for seller's welfare.[7]

Operationally, we identify outcomes that maximise the weighted sum of the ex ante *trade-surplus* of $\mathcal{S}$ and the *payoff* of $\mathcal{B}$ for arbitrary weights (see Holmström and Myerson (1983)). Thus, let $\gamma \in [0,1]$ be the Pareto weight on buyer surplus and $1-\gamma \in [0,1]$ be the weight on seller's trade surplus. We shall say an outcome is $\gamma-$*maximal* if it maximises the above $\gamma$-convex combination among the set of contractable-implementable outcomes without production subsidy.

When $F$ is regular, define the monotonically non-decreasing $\gamma$-*virtual valuation* as

$$\psi_{\mathcal{B},\gamma}(v) = v - \max\left\{0, \frac{1-2\gamma}{1-\gamma}\right\} \cdot \frac{1-F(v)}{f(v)}.$$

The main result of this section can be now stated.

**Theorem 2** *Assume $F$ is regular. For $\gamma \in [0,1]$, the unique $\gamma$-maximal outcome is:*

1. $q^*_\gamma(c,v) = \mathbb{1}\{c \leq \psi_{\mathcal{B},\gamma}(v)\}$;
2. $t^*_\gamma(c,v) = \mu_\gamma(v) \cdot \mathbb{1}\{c \leq \psi_{\mathcal{B},\gamma}(v)\}$, *where*

$$\mu_\gamma(v) := \begin{cases} \mathbb{E}_c[\psi^{-1}_{\mathcal{B},\gamma}(c) \mid c \leq \psi_{\mathcal{B},\gamma}(v)]^{8} & \text{if } \gamma \leq 1/2, \\ \mathbb{E}_c[c \mid c \leq v] & \text{if } \gamma > 1/2 \end{cases}$$

*with $\mathbb{E}[x \mid \emptyset] = 0$ is the expected payment of the $v$-buyer conditional on trade.*[9]

*The $\gamma$-maximal outcome is contract-implemented by a deterministic royalty contract*

$$k^*_\gamma(p) := \begin{cases} p - \psi_{\mathcal{B},\gamma}(\mu^{-1}_\gamma(p)) & p \in \mu_\gamma([\underline{v},\overline{v}]), \\ \overline{v} & \text{otherwise,} \end{cases}$$

*uniquely defined for $p \in \mu_\gamma([\underline{v},\overline{v}])$, together with any fixed fee $T^*_\gamma \in \mathbb{R}$.*

In the $\gamma$-optimal outcome, exchange takes place whenever the $\gamma-$virtual valuation of the buyer exceeds the seller's cost. To achieve this, the royalty payment provides a subsidy (or a surcharge) to the price $\mu_\gamma(v)$ posted in equilibrium by a value $v$ buyer, that makes just sellers with cost less than $\psi_{B,\gamma}(v)$ accept the offer. The price posted by the buyer can be understood as the equilibrium bid of a single bidder in a first-price auction with a random reserve price set at $\mu_\gamma(\psi^{-1}_{B,\gamma}(c))$.

Intuitively, the third party contract shapes the pricing strategy of the buyer by altering the distribution of the total cost that drives acceptance of the seller. Practically, it does so using two tools. First, it commits the seller to only accept a limited set of

[7]Ex ante, trade surplus equals profit if implementation is with budget balance. We do not focus on seller's profit, because a fixed transfer between the seller and the third-party does not affect the constraints of Lemma 1 nor (S-IR). Hence, for any achievable ex ante buyer payoff, any seller profit level can be attained if we do not restrict such transfer.

[8]Since we make no support assumptions, for a monotone mapping $[\underline{v},\overline{v}] \ni v \mapsto \ell(v) \in \mathbb{R}$, we employ the generalised inverse mapping $\mathbb{R} \ni x \mapsto \ell^{-1}(x) = \inf\{v \in [\underline{v},\overline{v}] \mid \ell(v) > x\} \in [\underline{v},\overline{v}]$.

[9]To be precise, at $\gamma = 1/2$, the payment is not unique as also $\mu_{\frac{1}{2}} - \overline{\mu}$ forms part of a $\gamma$-maximal outcome for any $\overline{\mu} \in \big[0, \mathbb{E}_c[\max\{0, \underline{v} - c\}]\big]$.

prices by prescribing unaffordable payments in case a price outside that set is accepted. Second, it engages in screening of the buyer by offering an incentive compatible menu of price-probability pairs.[10]

**Example 1** (Uniform distributions) *We characterise the ex ante Pareto frontier when* $v \sim U[0,1]$ *and* $c \sim U[0,1]$*. Define* $\Gamma = \max\{0, (1-2\gamma)/(1-\gamma)\}$*, so that* $\psi_{\mathcal{B},\gamma}(v) = (1+\Gamma)v - \Gamma$*. By Theorem 2, buyers with value* $v < \frac{\Gamma}{1+\Gamma}$ *never trade in the* $\gamma$*-maximal outcome. For* $v \geq \frac{\Gamma}{1+\Gamma}$*,* $p_\gamma(v) = \mu_\gamma(v) = \frac{v}{2} + \frac{\Gamma}{2(1+\Gamma)}$*. For* $p \in \mu_\gamma([0,1]) = \left[\frac{\Gamma}{1+\Gamma}, \frac{1+2\Gamma}{2(1+\Gamma)}\right]$ *we have* $k^*_\gamma(p) = p - \psi_{\mathcal{B},\gamma}(\mu_\gamma^{-1}(p)) = 2\Gamma - (1+2\Gamma)p$*.*

The royalty schedule $k^*_\gamma$ is uniquely defined for prices offered on path *and* accepted with some probability. The fixed fee, $T^*_\gamma$, is unrestricted since it affects neither trade-surplus nor behavior. However, in applications it is reasonable to bound $T^*_\gamma$ so that neither the seller nor the third-party incur an expected loss individually, not just jointly. At minimum $T^*_\gamma$ should balance the budget. At its maximum, the seller should break-even. That is, $-K^*_\gamma \leq T^*_\gamma \leq \hat{\Pi}(q^*_\gamma, t^*_\gamma) - K^*_\gamma$, where

$$K^*_\gamma := \mathbb{E}_{v,c}\left[q^*_\gamma(c,v)k^*_\gamma(\mu_\gamma(v))\right]$$

is the expected royalty payment.

Let $W^e := \mathbb{E}_{v,c}[\mathbb{1}_{\{v \geq c\}}(v-c)] \geq 0$ be the maximum expected gain from trade, that is the sum of ex ante buyer payoff and trade-surplus in the ex post efficient allocation. The following is also worth noting about the optimal royalty schedule $k^*_\gamma$.

**Corollary 2.1** *(i)* $K^*_\gamma \in [-W^e, 0]$*; (ii) If* $G$ *is regular then* $k^*_\gamma(p)$ *is strictly decreasing over prices* $p \in \mu_\gamma([\underline{v}, \overline{v}])$*; (iii) For* $1/2 > \gamma > \gamma'$ *and* $p \in \mu_\gamma([\underline{v}, \overline{v}]) \cap \mu_{\gamma'}([\underline{v}, \overline{v}])$*,* $k^*_\gamma(p) < k^*_{\gamma'}(p)$*; (iv) If* $G$ *is regular and* $[\underline{c}, \overline{c}] \cap \psi_{\mathcal{B},\gamma}([\underline{v}, \overline{v}])$ *has non-empty interior, then* $k^*_\gamma(p) < 0$ *if and only if* $p \in (\tilde{p}, \mu_\gamma(\overline{v})]$ *for some* $\tilde{p} < \mu_\gamma(\overline{v})$*.*

Part (i) bounds the expected royalty payment across the Pareto frontier. As generally $W^e > 0$, achieving a $\gamma-$maximal outcome requires royalty payments that, on average and without considering the fixed fee, transfer money to the seller. Part (ii) states that if $G$ is regular, then the optimal royalty is decreasing over prices which are posted and accepted with strictly positive probability. Part (iii) shows that if $\gamma'$ rises to $\gamma$, for any price posted on-path in both the $\gamma$- and $\gamma'$-maximal royalty schemes, royalties are necessarily decreasing on that price. That is, the supplier is subsidised more for accepting trade at each price offer. Part (iv) asserts that, with the exception of cases where the supports of cost and value are too far apart, the optimal scheme involves both positive and negative royalties. A positive payment dissuades the seller from accepting lower price offers, while a negative one encourages acceptance of higher prices.

We conclude this section by specialising Theorem 2 to the case in which asymmetric information is one sided, in the sense that either the valuation or the cost is known.

**Corollary 2.2** *Assume either* $F$ *is degenerate on* $v \in [\underline{v}, \overline{v}]$ *(known valuation) or* $G$ *is degenerate on* $c \in [\underline{c}, \overline{c}]$ *(known cost) and* $F$ *is regular. Then, the* $\gamma-$*maximal outcome*

[10]Sellers with increasingly high marginal cost are called upon to serve additional demand, the cost of providing higher probability of trade is convex. Hence, our optimal screening problem is isomorphic to that in a generalised version of Mussa and Rosen (1978) accounting for the different Pareto weights.

*is contract-implemented by a deterministic royalty scheme*

$$k_\gamma^*(p) = \begin{cases} \overline{v} & \textit{if } p \neq p_\gamma^* \\ -s_\gamma^* & \textit{if } p = p_\gamma^*. \end{cases}$$

*where*

$$(p_\gamma^*, s_\gamma^*) = \begin{cases} (\psi_{\mathcal{B},\gamma}^{-1}(c), 0) & \textit{if known cost,} \\ (v, 0)\mathbb{1}_{\{\gamma \leq 1/2\}} + (\mathbb{E}_c[c \mid c \leq v], v - \mathbb{E}_c[c \mid c \leq v])\mathbb{1}_{\{\gamma > 1/2\}} & \textit{if known value.}^{11} \end{cases}$$

When either cost or value is known, $\mathcal{B}$ is forced to choose between either offering $p_\gamma^*$ and trading with some probability or never trading. To do this, the implementing $k_\gamma^*$ demands the seller pay prohibitively high liquidated damages if it ever accepts trade at any other price, and subsidises trade at $p_\gamma^*$ by a payment of $s_\gamma^*$. Aside from the case of known value and $\gamma > 1/2$, $s_\gamma^* = 0$, implying the observed ubiquity of non-zero royalty schemes in Theorem 2 and Corollary 2.1 was an artefact of bilateral asymmetric information rather than unilateral.

## 4.1 Seller- and Buyer-Optimal outcomes

**Maximising Seller's surplus.** The outcome that maximises the seller's ex ante trade-surplus among all contract-implementable outcomes is the $0-$*maximal*. In such an outcome, trades takes place whenever

$$c \leq \psi_{\mathcal{B},0}(v) = v - \frac{1 - F(v)}{f(v)}.$$

It is well known that this is the allocation arising in the *monopoly-pricing game* where a privately informed seller makes a take-it-or-leave-it price offer to a single privately informed buyer with regular value distribution $F$. Because the minimum buyer payment is $\mu_\gamma(\underline{v}) \geq \underline{v}$, the lowest-type buyer always expects zero surplus. By the Payoff Equivalence Theorem (see Myerson (1981)), the expected revenue of a $c$-cost seller and the expected payoff of a $v$-value buyer in the outcome of the monopoly-pricing game are identical to those arising in the $0-$maximal outcome. As shown by Williams (1987), the seller's profit in the monopoly pricing game is the highest ex ante profit a seller can obtain across all equilibria of all selling mechanisms which guarantee each buyer type a payoff of at least zero. Summing up, we conclude the following.

**Corollary 2.3** *The expected trade-surplus in the trade-surplus maximising* ($0$-*maximal*) *outcome is the highest profit a seller can obtain in any equilibrium of any selling mechanism that satisfies interim buyer's participation.*

Remarkably, ex ante contracting between a third party and a seller who then trades with a buyer with monopoly power, can turn the buyer's monopoly into a seller's one.

Further, note that $K_0^* = 0$ and, thus, the budget-balancing $T_\gamma^*$ is also zero. In principle, a simple royalty scheme which makes the third-party break-even is sufficient

[11] As in Theorem 2, the 1/2-maximal outcome is non-unique: any $s_{1/2}^* \in [0, v - \mathbb{E}_c[c \mid c \leq v]]$ is 1/2-maximal when value is known.

to fully reverse bargaining power in the trading phase. Moreover, Corollary 2.1 implies that — besides where cost and value supports are sufficiently disjoint so trade either occurs almost surely or never — the royalty paid will be negative when the seller accepts high prices and positive when the seller accepts low prices. Intuitively, the optimal contract subsides $\mathcal{S}$ when higher prices are offered to make some high-cost types trade when they otherwise would not, in order to encourage $\mathcal{B}$ to post those higher prices. Conversely, acceptance at lower prices is penalised to reduce the probability of trade at those price as a means of encouraging the buyer not to post lower prices.

**Example 1.2** (Uniform distributions (continued)) *At the seller-optimal* ($\gamma = 0$) $\Gamma = 1$, *so types* $v < 1/2$ *are excluded from trade and types* $v \geq 1/2$ *post* $p_0(v) = \frac{v}{2} + \frac{1}{4}$. *The royalty schedule is* $k_0^*(p) = 2 - 3p$, *meaning that pair* $(c, v)$ *trade if and only if* $c \leq p_0(v) - k_0^*(p_0(v)) = 2v - 1$.

**Maximising Buyer-Surplus** In the $1-$*maximal* outcome (and for any $\gamma \geq 1/2$), which maximises the buyer's ex ante surplus, trades takes place whenever it is *ex post efficient*, that is when

$$c \leq \psi_{\mathcal{B},1}(v) = v.$$

Moreover, the buyer extracts the entire first-best surplus $W^e$. In fact

$$\mathbb{E}_{v,c}\Big[q_\gamma^*(c,v)v - t_\gamma^*(c,v)\Big] = \mathbb{E}_{v,c}\Big[\mathbb{1}\{c \leq v\}(v - \mathbb{E}_c[c \mid c \leq v])\Big] = \mathbb{E}_{v,c}[\mathbb{1}_{\{v \geq c\}}(v - c)].$$

To understand the result, note that a buyer with value $v$ offers $\mu_\gamma(v) = \mathbb{E}_c[c \mid c \leq v]$, a price that exactly compensates $\mathcal{S}$ for the expected cost it sustains if it sells whenever $c \leq v$. In turn, the royalty is equal to the price posted by $\mathcal{B}$ minus the value of $\mathcal{B}$ inferred from the price just offered to $\mathcal{S}$, in such a way that $\mathcal{S}$ earns $v - c$ if it accepts the offer from a $v$-type buyer, no matter its cost. That is, both the seller's profit gross of the fixed fee and the buyer surplus are equal to the entire surplus generated in the downstream market. Nevertheless, both the third-party and $\mathcal{S}$ break even ex ante, because the former recovers via the fixed fee the average payment it needed to pay to $\mathcal{S}$ as subsidy, which is equal to the first best-welfare $W^e$.

The buyer-optimal, budget-balanced outcome can be interpreted as a direct mechanism with transfers for both the buyer and the seller, $(\tilde{q}, \tilde{t}_B, \tilde{t}_S)$, where $\tilde{t}_S$ is the effective transfer to seller including royalty payments that implements the ex post efficient allocation of the good between a buyer and seller, $\tilde{q}(c,v) = \mathbb{1}\{c \leq v\}$. The ex post payment of the buyer is cost-independent and equal, in every state, to the expected externality it imposes on the seller given their value, that is $\tilde{t}_B(c,v) = \mathbb{1}\{c \leq v\}\mathbb{E}_c[c \mid c \leq v]$, as in d'Aspremont and Gérard-Varet (1979). The ex post payment of the seller takes the Groves (1973) form: the seller is made residual claimant on the realised surplus (i.e., if $c \leq v$ it receives a transfer of $\mu_1(v) - k_1^*(\mu_1(v)) = v$ and bears cost $c$); in addition, it pays a fixed fee, independent of both cost and value, equal to the ex ante surplus in the efficient outcome, $W^e$. Hence, $\tilde{t}_S(c,v) = \mathbb{1}\{c \leq v\}v$ The resulting mechanism is Bayesian incentive compatible and interim individually rational for the buyer and dominant-strategy incentive compatible for the seller. It is budget balanced as $\mathcal{S}$ pays a fixed fee of $W^e$ and

$$\mathbb{E}_{v.c}\Big[\tilde{t}_{\mathcal{S}}(c,v) - \tilde{t}_{\mathcal{B}}(c,v)\Big] = \mathbb{E}_{v.c}\Big[\mathbb{1}\{c \leq v\}v - \mathbb{1}\{c \leq v\}\mathbb{E}_c[c \mid c \leq v]\Big] = W^e.$$

The impossibility result of Myerson and Satterthwaite (1983) is circumvented because only ex ante (rather than interim) individual rationality of the seller is satisfied.

**Example 1.3** (Uniform distributions (continued)) *At the buyer-optimal* ($\gamma = 1$) $\Gamma = 0$, *so every buyer-type* $v > 0$ *trades with strictly positive probability and posts price* $p_1(v) = \frac{v}{2}$. *The royalty schedule is* $k_1^*(p) = -p$, *meaning that pair* $(c, v)$ *trade if and only if* $c \leq p_1(v) - k_1^*(p_1(v)) = v$.

## 4.2 Welfare Impact of Contracting

Here, we compare the total welfare — the sum of buyer and seller surplus — in the case of *no ex ante contracting* with each $\gamma$-maximal outcome for $\gamma \in [0, 1]$ as characterised in Theorem 2.

As a first step, let's define the outcome in the absence of ex ante contracting. The buyer picks the price schedule $p^{\mathcal{B}}(v)$, generating total welfare $W_{\mathcal{B}}$, where

$$p^{\mathcal{B}}(v) \in \arg\max_{p} G(p)(v - p), \qquad W^{\mathcal{B}} := \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{p^{\mathcal{B}}(v) \geq c\}}(v - c) dG dF.$$

Moreover, denote by $W_\gamma$ the total welfare in the $\gamma$-maximal outcome,

$$W_\gamma := \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{\psi_{\mathcal{B},\gamma}(v) \geq c\}}(v - c) dG dF.$$

We know $W_0 \leq W_1$, given $W_1 = W^e$. The next proposition shows total welfare in the $\gamma$-maximal outcome is weakly monotone increasing in $\gamma$.

**Lemma 3** *For any* $\gamma, \gamma' \in [0, 1]$,

$$\gamma < \gamma' \implies W_\gamma \leq W_{\gamma'}$$

This Lemma allows us to prove the main result of this subsection. We show that if the seller's monopoly-pricing outcome generates higher welfare than the buyer's monopoly-pricing outcome, the total welfare effect of ex ante contracting is positive across the ex ante frontier. Conversely, if a lower welfare is generated, the welfare effect of ex ante contracting is ambiguous across the frontier.

**Proposition 1** *If* $W_0 > W_{\mathcal{B}}$ *then* $W_\gamma > W_{\mathcal{B}}$ *for all* $\gamma \in [0, 1]$. *If* $W_0 < W_{\mathcal{B}}$, *there exists unique* $\gamma^* \in (0, 1]$ *such that* $W_\gamma < W_{\mathcal{B}}$ *for* $\gamma \in [0, \gamma^*)$ *and* $W_\gamma \geq W_{\mathcal{B}}$ *for* $\gamma \in [\gamma^*, 1]$.

The following example elaborates on the case in which both cost and value are uniformly distributed.

**Example 1.4** (Uniform distributions (continued)) *With no contracting* ($k \equiv 0$), *the buyer posts price* $p_{\mathcal{B}}(v) = v/2$ *and trade occurs if and only if* $c \leq v/2$, *giving* $W^{\mathcal{B}} = 1/8$. *Given earlier calculations,* $W_\gamma = \frac{2\Gamma+1}{6(\Gamma+1)^2}$. *Cost and value distribution symmetry imply* $W^{\mathcal{B}} = W_0$, *In Figure 2, we plot the total welfare in the* $\gamma$*-maximal outcome versus the total welfare in the case of no ex ante contracting.*

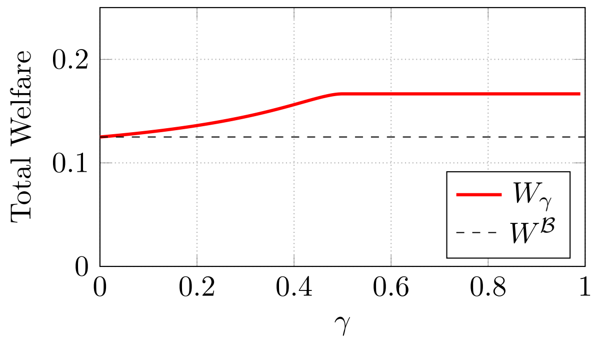



**Figure 2:** Total welfare in $\gamma$-maximal outcome versus in case of no ex ante contracting.

## 4.3 Relation to Myerson and Satterthwaite (1983)

In Myerson and Satterthwaite (1983), the search for an ex post efficient mechanism is restricted to the set of direct mechanisms that satisfy Bayesian incentive compatibility and interim individual rationality for both buyer and seller. If an outcome satisfies these conditions, we shall say the outcome is *MS-implementable*.

Relative to the conditions for contract implementation without production subsidy, MS-implementation weakens the seller's constraints from monotonicity in cost of the interim allocation to monotonicity of the ex post one, which characterises dominant strategy incentive compatibility, and strengthens the seller's participation constraints from ex ante to ex-interim. The latter turns out to be decisive, perhaps not surprisingly in light of Manelli and Vincent (2010) and subsequent work by Gershkov et al. (2013) on the equivalence between Bayesian incentive compatibility and dominant strategy incentive compatible mechanisms.

**Proposition 2** *Let $\mathbb{U}_{MS}, \mathbb{U}_C$ be the set of MS-implementable and contract-implementable ex ante utility profiles, respectively. $\mathbb{U}_{MS} \subseteq \mathbb{U}_C$, with strict nesting whenever cost and value supports overlap.*

In particular, all ex ante Pareto efficient profiles in $\mathbb{U}_{MS}$ can be attained through budget-balanced ex ante contracting. The proof proceeds by showing that a class of mechanisms spanning $\mathbb{U}_{MS}$, viz. randomisations of markup-pooling mechanisms (Proposition 9 of Yang and Yang (2025)), are ex ante equivalent to some contract-implementable outcome.[12]

**Example 1.5** (Uniform distributions (continued)) *In Figure 3, we show how ex ante contracting expands the utility possibility set beyond those that are MS-implementable. See Appendix B for computations.*

[12] In markup-pooling mechanisms, trade occurs if and only if the value of the buyer is above a monotone transform of the seller's cost (the markup function), with the exception that when cost falls into a single interval (the pooling interval) the cost is resampled from the two ends of the interval. A formal definition is provided in Appendix A.4.

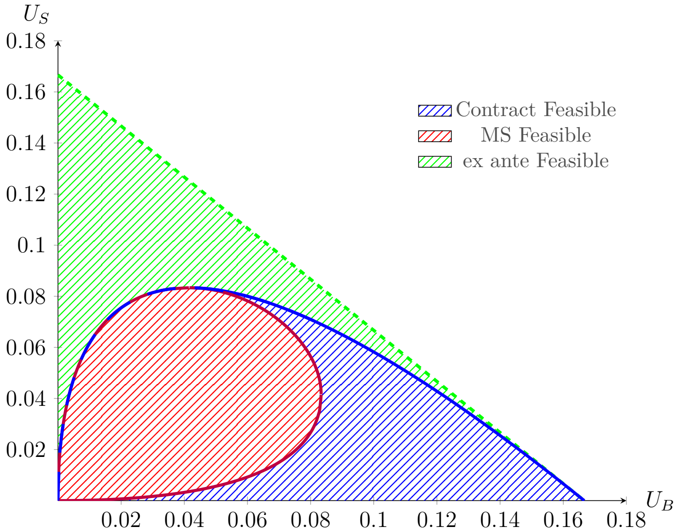



**Figure 3:** Ex ante buyer-seller utility possibility sets $(U_B, U_S)$ — uniform case

## 5 Discussion of Assumptions

**Renegotiation Proofness** We deliberately abstracted from any preferences of the third-party, focusing instead on the set of outcomes implementable through contracting. In practice, however, a third party may have its own objective. In that case, after observing either the seller's cost or the buyer's posted price, the third party and the seller may find it mutually beneficial to renegotiate.

First, suppose the third-party is welfare-maximising and the contract is force is the 1-maximal outcome of Theorem 2, which achieves ex post efficient trade. The following is obvious.

**Remark 1** *If the third party is a welfare maximiser, no profitable renegotiation exists away from the buyer-optimal contract.*

Instead, assume the third party is revenue maximiser. In this case the third-party will wish to implement the 0-maximal outcome and extract the seller surplus via the fixed fee. Two natural renegotiation opportunities arise: (i) after the buyer posts its price, and (ii) after the seller's cost is realised but before the buyer's price is posted.

Suppose renegotiation is possible after $\mathcal{B}$ posts its price, such that some cost types would benefit from trade. If the buyer cannot post a new price after observing renegotiation, the third party and $\mathcal{S}$ would renegotiate to the null contract ($k^* \equiv 0$), allowing the seller to trade whenever $c \leq p$. If $k(p) > 0$, cost types $c \in (\max\{\underline{c}, p - k(p)\}, p)$ strictly benefit from the removal of royalty payments. If $k(p) < 0$, the third party's revenue is improved by renegotiation to the null royalty. In either case, the gains may be split between third party and $\mathcal{S}$. If the buyer is aware such renegotiation is possible — regardless of whether they observe the renegotiated contract — they ignore the ex ante contract and post their monopoly price. The resulting outcome is equivalent to that arising when $\mathcal{B}$ has complete bargaining power. Therefore, renegotiation following the posting of a price causes the countervailing power of the 0-maximal contract to collapse entirely.

Renegotiation following cost realisation but before price-posting is more delicate. In fact, the following is true since we showed the 0-maximal outcome upper bounded the seller's ex ante profit across all outcomes satisfying buyer individual rationality and incentive compatibility.

**Remark 2** *An ex ante contract that is contingent on the cost realisation cannot increase the revenue of the third party contractor beyond that achieved in Corollary 2.3.*

Yet, there might be incentives to renegotiate if certain cost values realise. Assuming $\mathcal{B}$ observes the new contract, there may be interim gains from renegotiating from the seller-optimal royalty contract $k_0^*$ to a contract which stipulates 0 royalty fees for trade at a price equal to $p^{\mathcal{S}}(c)$, where $p^{\mathcal{S}}(c) \in \arg\max_p (p-c)(1-F(p))$, and charges extremely high royalties should the seller accept any other price. As in Corollary 2.2, the new contract enforces that the buyer chooses between either posting $p^{\mathcal{S}}(c)$ and trading, or forging all trade. To prove this observation we invoke the following result.

**Lemma 4** *All cost types of $\mathcal{S}$ obtain the same trade surplus under a contract with royalty scheme $k_0^*$ and a $p^{\mathcal{S}}(c)$-forcing contract.*

*Proof.* From integration by parts,

$$\begin{aligned}\int_{\underline{v}}^{\overline{v}} \mathbb{1}_{\{\psi_{\mathcal{B},0}(v)\geq c\}}(\psi_{\mathcal{B},0}(v)-c)dF &= \int_{\underline{v}}^{\overline{v}} \mathbb{1}_{\{\psi_{\mathcal{B},0}(v)\geq c\}}[(v-c)f(v)-(1-F(v))]dv \\ &= (\psi_{\mathcal{B},0}^{-1}(c)-c)(1-F(\psi_{\mathcal{B},0}^{-1}(c))) \\ &= \int_{\underline{v}}^{\overline{v}} \mathbb{1}_{\{\psi_{\mathcal{B},0}(v)\geq c\}}(\psi_{\mathcal{B},0}^{-1}(c)-c)dF\end{aligned}$$

First expression is seller's payoff under $k_0^*$ and last expression is their payoff under a $p^{\mathcal{S}}(c)-$forcing contract. □

Lemma 4 implies that, taking the fixed fee as sunk, cost types above the threshold $\tilde{p}$ defined in Corollary 2.1 — who expect to receive a royalty payment — are indifferent between sticking to the 0-maximal contract and transitioning to an observable $p^{\mathcal{S}}(c)$-forcing contract in exchange for forgoing the positive royalty payment they expect to receive. Such a move strictly raises the third party's revenue as they are no longer obligated to pay out negative royalties to the seller.[13] We conclude that for an interval of high cost types renegotiation to a Pareto improving outcome is conceivable if $\mathcal{S}$ has signed the 0-maximal contract.

**Observability of Contracting** Throughout the paper, we assumed the buyer observed the contract signed by $\mathcal{S}$. We now discuss the case in which the contract between the third party and the seller is not observable by the buyer.

Because, by Remark 1, the third party has no incentive to sign a different contract if they are a welfare maximiser, we focus on how Corollary 2.3 is affected by such non-observability when the third party is a revenue maximiser.

[13] It follows from this observation that some cost types obtain higher trade surplus under a contract $(0, k^*)$ than in the seller-optimal mechanism where $\mathcal{S}$ is a monopolist. The existence of mechanisms that give some type of sellers a payoff higher than the monopoly payoff is demonstrated by example in Yilankaya (1999)

**Proposition 3** *Suppose $\mathcal{B}$ cannot observe the third-party contract. Then the seller-optimal royalty contract is the null scheme, $k^* \equiv 0$, with the resulting trade surplus of $\mathcal{S}$ being extracted via the fixed fee.*

The reasoning for Proposition 3 is straightforward from our discussion of renegotiation proofness and the proof is omitted.

**Contractibility Assumptions** Throughout the paper, we assumed the third-party contract could condition on whether trade occurs and what price the buyer posts to the seller.

If the contract can only condition on what price the buyer posts, then any payments required from the seller are sunk by the time the seller is deciding whether to accept trade with the buyer at the posted price or not. Consequently, the contract cannot implement anything beyond the downstream mechanism arising when there is no contract.

**Remark 3** *If the third-party contract can condition on the price posted, but not on whether trade occurs or not, then the contract cannot affect the outcome.*

Instead, if the third-party contract can condition on whether trade occurs but not on the price posted, the downstream mechanism may be influenced by the contract. In what follows, we shall close down the possibility of randomisation in the contract. We next define the associated restricted form of implementation.

**Definition 3** *An outcome $(q, t)$ is* flat-contract implementable *if $(q, t)$ is contract implementable by a deterministic contract $m$ which is constant over prices, $p$.* [14]

Recall, $p^{\mathcal{B}}(v)$ represents the price schedule of the buyer when the null contract is in place ($m \equiv 0$). With this, the next Lemma characterises the flat-contract-implementable outcomes.

**Lemma 5** *$(q, t)$ is a flat-contract-implementable outcome if and only if:*

1. *$q(c, v) = \mathbb{1}\{p^{\mathcal{B}}(v - \overline{k}) \geq c\}$ for some $\overline{k} \in \mathbb{R}$.*
2. *$(q, t)$ is contract implementable*

The restriction imposed by Condition (1) in Lemma 5 is non-trivial. Without being able to write a contract contingent on the price at which trade occurs, the contract cannot alter the buyer's price schedule beyond effectively shifting the buyer's value downwards by a fixed constant.

*Proof.* As a flat contract is a type of contract, any flat-contract-implementable outcome is necessarily contract implementable. Further, if the fee demanded when trade occurs is $\overline{k} \in \mathbb{R}$, the seller accepts to trade if and only if $p - \overline{k} \geq c$. So, the buyer's optimal price schedule solves

$$p_{\overline{k}}(v) \in \arg\max_p G(p - \overline{k})(v - p) \iff p_{\overline{k}}(v) = \overline{k} + p^{\mathcal{B}}(v - \overline{k})$$

[14] If we allow for contract randomisation by $\omega$, trade probabilities take the form $q(c, v) = \mathbb{P}_\omega\big(m_\omega \leq p(v) - c\big)$ for some random variable $m_\omega$ and the buyer's posted-price schedule $p(v)$. For deterministic outcomes ($q(c, v) \in \{0, 1\}$) it is without loss to restrict attention to degenerate $m_\omega$. Therefore, under $F$-regularity, Theorem 2 implies any point on the frontier is flat-contract implementable by some random $m_\omega$ only if it is also implementable by a degenerate flat contract.

The induced allocation rule is $q(c, v) = \mathbb{1}\{(\bar{k} + p^{\mathcal{B}}(v - \bar{k})) - \bar{k} \geq c\} = \mathbb{1}\{p^{\mathcal{B}}(v - \bar{k}) \geq c\}$.

For the reverse implication, if the two conditions hold then by Condition (2), the outcome is contract implementable. Then Condition (1) implies the outcome is further flat-contract implementable. □

Contrary to Corollary 2.3, the seller-optimal flat-contract-implementable outcome only partially countervails the bargaining power of $\mathcal{B}$, but is unable to implement the seller-optimal outcome. That flat-contract-implementable outcomes can do no better than contract-implementable outcomes is immediate from Lemma 5: flat contracts add an additional constraint on top of contract implementability.

We illustrate by means of a uniform example that flat-contract implementation may fail to implement the seller-optimal.

**Example 1.6** (Uniform distributions (continued)) *By Lemma 5, the contract choice boils down to a single value of $\bar{k}$. Let us consider the flat contract which optimises seller surplus. Utilising payoff equivalence, one derives that in any flat-contract-implementable outcome, the seller's surplus is equal to*

$$\int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\left\{\frac{v-\bar{k}}{2} \geq c\right\}} (2v - 1 - c) dG dF$$

*This is optimised at $\bar{k} = 0.2$, giving the seller a surplus of $\frac{4}{75}$. However, the seller-optimal outcome gives the seller a surplus equal to $\frac{1}{12}$. Because $\frac{1}{12} > \frac{4}{75}$ the example demonstrates that the seller-optimal outcome is generally not implementable when the contract can only condition on whether trade occurs and not on the price posted by the buyer.*

In any flat-contract-implementable outcome, trade occurs if and only if $c \leq p(v) - \bar{k} = p^{\mathcal{B}}(v - \bar{k})$. Because $p^{\mathcal{B}}$ is non-decreasing, raising $\bar{k}$ can only lower the trading efficiency in the downstream market relative to the scenario in which the third party offers $\mathcal{S}$ a null contract ($\bar{k} = 0$). We now show that even for $\bar{k} < 0$, ex post efficient trade is flat-contract implementable.

**Proposition 4** *Suppose $G$ is regular. Efficient trade is not flat-contract implementable whenever $[\underline{v}, \overline{v}] \cap [\underline{c}, \overline{c}] = [\underline{c}, \overline{v}]$ has strictly positive Lebesgue measure.*

*Proof.* By Lemma 5, for any flat-contract-implementable outcome trade occurs according to $q(c, v) = \mathbb{1}\{p_{\mathcal{B}}(v - \bar{k}) \geq c\}$ for some $\bar{k} \in \mathbb{R}$. Given $[\underline{v}, \overline{v}] \cap [\underline{c}, \overline{c}] = [\underline{c}, \overline{v}]$, for $v \in [\underline{c}, \overline{c}]$ efficient trade requires $v = p_{\mathcal{B}}(v - \bar{k})$, yet by $G$-regularity this requires

$$v + \frac{G(v)}{g(v)} = v - \bar{k} \iff \frac{G(v)}{g(v)} = -\bar{k} \implies \frac{G(v)}{g(v)} \text{ constant for } v \in [\underline{c}, \overline{c}]$$

We prove this is impossible. First, for $v \in (\underline{c}, \overline{c}]$, $G(v) > 0$ and $g(v) > 0$ so $\bar{k} < 0$. Rewrite the condition as $g(v) = \frac{-1}{\bar{k}} G(v)$. The solutions of this equation are $G(v) = Ae^{-v/\bar{k}}$ for constant $A \in \mathbb{R}$. Yet,

$$0 \xleftarrow{v \searrow \underline{c}} G(v) = Ae^{-v/\bar{k}} \xrightarrow{v \searrow \underline{c}} Ae^{-\underline{c}/\bar{k}}$$

giving $A = 0$. But this violates the properties of $G$ being a CDF. Hence, efficient trade is impossible. □

# APPENDIX

# A Omitted Proofs

Proofs are presented in the same order as in the text, except for a formula from Myerson and Satterthwaite (1983) that we shall use repeatedly which we state first.

**Lemma A.1** (Myerson and Satterthwaite (1983) formula) *For $F$ a regular value distribution,*

$$\int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{v \geq c\}} \left(\psi_{\mathcal{B},0}(v) - c\right) dGdF = \int_{\underline{c}}^{\overline{c}} (1 - F(x))G(x)dx - \int_{\underline{v}}^{\overline{c}} (1 - F(x))G(x)dx$$
$$= \int_{\underline{c}}^{\max\{\underline{c},\underline{v}\}} G(x)dx$$

*Proof.* Immediate from formulae on page 272 of Myerson and Satterthwaite (1983). □

## A.1 Proof of Lemma 2

*Proof.* Fix contract $m$ and $(p_m, a_m) \in \mathcal{E}(m)$. We decompose $m$ as

$$m(\omega, x, p) = x[m(\omega, 1, p) - m(\omega, 0, p)] + m(\omega, 0, p) = xk(\omega, p) + T(\omega, p)$$

As $T(\omega, p)$ is paid irrespective of whether $\mathcal{S}$ accepts trade, changes to $T(\omega, p)$ do not affect the outcome implemented by the contract. Therefore, so long as $\mathbb{E}_{\omega,v}[|T(\omega, p_m(v))|] < \infty$ we can construct a new contract $\tilde{m}(\omega, x, p) = xk(\omega, p) + \mathbb{E}_{\omega,v}[T(\omega, p_m(v))]$. That $\mathbb{E}_{\omega,v}[|T(\omega, p_m(v))|] < \infty$ follows immediately due to the uniform upper bound on the contract fees. As argued, since the fixed fee was sunk and hence does not affect seller or buyer's incentives, $(p_m, a_m) \in \mathcal{E}(\tilde{m})$. Further,

$$\mathbb{E}_{\omega,c,v,x\sim Bern(a(\omega,c,p_m(v)))}[\tilde{m}(\omega, x, p_m(v))] = \mathbb{E}_{\omega,c,v,x\sim Bern(a(\omega,c,p_m(v)))}[m(\omega, x, p_m(v))]$$

and so expected revenue for third party is equal under $m$ and $\tilde{m}$. □

## A.2 Proof of Theorem 2

*Proof.* We show $(q^*_\gamma, t^*_\gamma)$ achieves an upper bound on the $\gamma$-convex combination of buyer and seller surpluses across all outcomes satisfying the Conditions of Lemma 1 and (S-IR). Then, we show $(q^*_\gamma, t^*_\gamma)$ itself satisfies Lemma 1 and (S-IR).

*Upper Bound:* Fix $(q, t)$ a contract-implementable outcome satisifying (S-IR). Let $\hat{\Pi}_\gamma(q, t)$ be the $\gamma$-convex combination of buyer and seller trade surpluses under $(q, t)$. Using Fubini and the buyer's participation and incentive compatibility constraints, if

$\gamma < 1/2$,

$$\begin{aligned}
\hat{\Pi}_\gamma(q,t) &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \gamma[q(c,v)v - t(c,v)] + (1-\gamma)[t(c,v) - cq(c,v)] dGdF \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \begin{pmatrix} q(c,v)\left[\gamma(v - \psi_{0,\mathcal{B}}(v)) + (1-\gamma)(\psi_{0,\mathcal{B}}(v) - c)\right] \\ + \underbrace{(2\gamma - 1)}_{\leq 0} \underbrace{(q(c,\underline{v})\underline{v} - t(c,\underline{v}))}_{\geq 0} \end{pmatrix} dGdF \\
&\leq \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} q(c,v)\left(\gamma(v - \psi_{0,\mathcal{B}}(v)) + (1-\gamma)(\psi_{0,\mathcal{B}}(v) - c)\right) dGdF \\
&= (1-\gamma) \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} q(c,v) \left(v + \frac{2\gamma - 1}{1-\gamma} \frac{1-F(v)}{f(v)} - c\right) dGdF =: \overline{\Pi}_\gamma(q)
\end{aligned}$$

For $\gamma < 1/2$, $\hat{\Pi}_\gamma(q^*_\gamma, t^*_\gamma) \geq \overline{\Pi}_\gamma(q) \geq \hat{\Pi}_\gamma(q,t)$ from pointwise maximisation of the above.

For $\gamma > 1/2$, we instead have

$$\begin{aligned}
\hat{\Pi}_\gamma(q,t) &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \gamma[q(c,v)v - t(c,v)] + (1-\gamma)[t(c,v) - cq(c,v)] dGdF \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} (2\gamma - 1)[q(c,v)v - t(c,v)] + (1-\gamma)[q(c,v)(v-c)] dGdF \\
&= (1-\gamma) \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} [q(c,v)(v-c)] dGdF + (2\gamma - 1) \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} q(c,v)v - t(c,v) dGdF
\end{aligned}$$

By (S-IR) and noting $t(c,v) - q(c,v)c = q(c,v)(v-c) - (q(c,v)v - t(c,v))$ we require

$$\begin{aligned}
0 &\leq \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} t(c,v) - q(c,v)c \, dGdF \\
&\leq \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} q(c,v)(v-c) - (q(c,v)v - t(c,v)) \, dGdF \\
\int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} q(c,v)v - t(c,v) \, dGdF &\leq \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} q(c,v)(v-c) \, dGdF
\end{aligned}$$

Substituting into $\hat{\Pi}_\gamma(q,t)$ we obtain:

$$\begin{aligned}
\hat{\Pi}_\gamma(q,t) &= (1-\gamma) \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} [q(c,v)(v-c)] dGdF + (2\gamma - 1) \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} q(c,v)v - t(c,v) dGdF \\
&\leq (1-\gamma) \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} [q(c,v)(v-c)] dGdF + (2\gamma - 1) \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} q(c,v)(v-c) dGdF \\
&\leq \gamma \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} q(c,v)(v-c) \, dGdF \\
&\leq \gamma \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{v \geq c\}}(v-c) \, dGdF := \bar{\Pi}_\gamma
\end{aligned}$$

With $\gamma > 1/2$ the posited $\gamma$-maximal allocation and transfer rules are $q^*_\gamma = \mathbb{1}_{\{c \leq v\}}$, $t^*_\gamma = \mu_\gamma(v)\mathbb{1}_{\{c \leq v\}}$. We show that $\hat{\Pi}_\gamma(q^*_\gamma, t^*_\gamma) = \bar{\Pi}_\gamma$ by demonstrating that in $(q^*_\gamma, t^*_\gamma)$, the seller trade surplus is 0, while the buyer's ex ante surplus is equal to the total value of efficient trade.

First, the seller makes 0 trade surplus since,

$$\begin{aligned}\int_{\underline{v}}^{\overline{v}}\int_{\underline{c}}^{\overline{c}}\Big(t^*_\gamma(c,v)-c\,q^*_\gamma(c,v)\Big)dGdF &= \int_{\underline{v}}^{\overline{v}}\int_{\underline{c}}^{\overline{c}}\Big(\mu_\gamma(v)\mathbb{1}_{\{c\le v\}}-c\,\mathbb{1}_{\{c\le v\}}\Big)dGdF\\ &= \int_{\underline{v}}^{\overline{v}}\int_{\underline{c}}^{\overline{c}}\mathbb{1}_{\{c\le v\}}\Big(\mu_\gamma(v)-c\Big)dGdF\\ &= \int_{\underline{v}}^{\overline{v}}\left[G(v)\mu_\gamma(v)-\left(\int_{\underline{c}}^{\overline{c}}1_{\{c\le v\}}cdG\right)\right]dF\\ &= \int_{\underline{v}}^{\overline{v}}\left[G(v)\mu_\gamma(v)-G(v)\mathbb{E}_c[c\mid c\le v]\right]dF\\ &= 0.\end{aligned}$$

Second, the buyer's ex ante payoff is the total value of efficient trade because,

$$\begin{aligned}\int_{\underline{v}}^{\overline{v}}\int_{\underline{c}}^{\overline{c}}\Big(q^*_\gamma(c,v)v-t^*_\gamma(c,v)\Big)dGdF &= \int_{\underline{v}}^{\overline{v}}\int_{\underline{c}}^{\overline{c}}\mathbb{1}_{\{v\ge c\}}(v-\mu_\gamma(v))dGdF\\ &= \int_{\underline{v}}^{\overline{v}}G(v)(v-\mu_\gamma(v))dF\\ &= \int_{\underline{v}}^{\overline{v}}vG(v)-G(v)\mathbb{E}_c[c\mid c\le v]dF\\ &= \int_{\underline{v}}^{\overline{v}}vG(v)-\left(\int_{\underline{c}}^{\overline{c}}\mathbb{1}_{\{v\ge c\}}cdG\right)dF\\ &= \int_{\underline{v}}^{\overline{v}}\int_{\underline{c}}^{\overline{c}}\mathbb{1}_{\{v\ge c\}}(v-c)dGdF\end{aligned}$$

Hence, $\hat{\Pi}_\gamma(q^*_\gamma,t^*_\gamma)=\bar{\Pi}_\gamma\ge\Pi(q,t)$ for any $(q,t)$ which is contract-implementable without outside subsidy when $\gamma>1/2$.

Therefore, if $(q,t)$ is contract-implementable and satisfies (S-IR), $\hat{\Pi}_\gamma(q,t)\le\hat{\Pi}_\gamma(q^*_\gamma,t^*_\gamma)$.

*Implementation* We verify that $(q^*_\gamma,t^*_\gamma)$ satisfies the conditions of Lemma 1. First, it is clearly of pure posted price form, so Condition 5 is satisfied. Because $q^*_\gamma(c,v)=\mathbb{1}\{\psi_{\mathcal{B},\gamma}(v)\ge c\}$, $q^*_\gamma(c,v)$ is clearly non-increasing in $c$ for each fixed $v$, giving Condition 1.

The second condition of Lemma 1 holds because $\psi_{\mathcal{B},\gamma}(v)$ is increasing by the $F$-regularity assumption and since $\int_{\underline{c}}^{\overline{c}}q(c,v)dG=G(\psi_{\mathcal{B},\gamma}(v))$.

For $\gamma<1/2$ consider, $\mu_\gamma(\underline{v})=\mathbb{E}_c[\psi^{-1}_{\mathcal{B},\gamma}(c)\mid\psi_{\mathcal{B},\gamma}(\underline{v})\ge c]=\underline{v}$. Therefore,

$$\int_{\underline{c}}^{\overline{c}}q^*_\gamma(c,\underline{v})\underline{v}-t^*_\gamma(c,\underline{v})dG=\int_{\underline{c}}^{\overline{c}}\mathbb{1}_{\{\psi_{\mathcal{B},\gamma}(v)\ge c\}}[\underline{v}-\mu_\gamma(\underline{v})]dG=0$$

On the other hand, if $\gamma>1/2$,

$$\int_{\underline{c}}^{\overline{c}}q^*_\gamma(c,\underline{v})\underline{v}-t^*_\gamma(c,\underline{v})dG=\int_{\underline{c}}^{\overline{c}}\mathbb{1}_{\{\underline{v}\ge c\}}(\underline{v}-\mathbb{E}_c[c\mid\underline{v}\ge c])dG\ge 0$$

Therefore, Condition 4 is satisfied for any $(q^*_\gamma, t^*_\gamma)$, $\gamma \in [0,1]$.

Finally, we show that

$$\int_{\underline{c}}^{\overline{c}} t^*_\gamma(c,v)dG = \int_{\underline{c}}^{\overline{c}} \left[ vq^*_\gamma(c,v) - \underline{v}q^*_\gamma(c,\underline{v}) + t^*_\gamma(c,\underline{v}) - \int_{\underline{v}}^{v} q^*_\gamma(c,x)dx \right] dG$$

Note that buyer's interim transfers are $\int_{\underline{c}}^{\overline{c}} t^*_\gamma(c,v)dG = \mu_\gamma(v)G(\psi_{\mathcal{B},\gamma}(v))$.

Suppose $\gamma < 1/2$, then lowest type buyer makes zero profit. For $v \in [\underline{v}, \overline{v}]$, $\psi_{\mathcal{B},\gamma}(v) \leq \underline{c}$ trade never occurs and no transfer is ever paid in $(q^*_\gamma, t^*_\gamma)$ so the envelope condition is trivially satisfied. For $v$-types with $\psi_{\mathcal{B},\gamma}(v) > \underline{c}$, then

$$\int_{\underline{c}}^{\overline{c}} \left[ vq^*_\gamma(c,v) - \int_{\underline{v}}^{v} q^*_\gamma(c,x)dx \right] dG = \underbrace{\left[ v - \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x))dx \right]}_{:=A(v)} G(\psi_{\mathcal{B},\gamma}(v)$$

Then,

$$\begin{aligned}
A(v) &= v - \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x))dx \\
&= v - \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \left[ [xG(\psi_{\mathcal{B},\gamma}(x))]_{\underline{v}}^{v} - \int_{\underline{v}}^{v} xdG(\psi_{\mathcal{B},\gamma}(x)) \right] \\
&= \frac{G(\psi_{\mathcal{B},\gamma}(\underline{v}))}{G(\psi_{\mathcal{B},\gamma}(v))}\underline{v} + \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \int_{\underline{v}}^{v} xdG(\psi_{\mathcal{B},\gamma}(x)) \\
&= \frac{G(\psi_{\mathcal{B},\gamma}(\underline{v}))}{G(\psi_{\mathcal{B},\gamma}(v))}\underline{v} + \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \int_{\psi_{\mathcal{B},\gamma}(\underline{v})}^{\psi_{\mathcal{B},\gamma}(v)} \psi^{-1}_{\mathcal{B},\gamma}(u)dG \\
&= \mathbb{E}_c[\psi^{-1}_{\mathcal{B},\gamma}(c) \mid c \leq \psi_{\mathcal{B},\gamma}(v)] = \mu_\gamma(v)
\end{aligned}$$

where again we recall we use the generalised inverse for $\psi_{\mathcal{B},\gamma}$. Therefore, when $\gamma \leq 1/2$ for all $v \in [\underline{v}, \overline{v}]$,

$$\int_{\underline{c}}^{\overline{c}} t^*_\gamma(c,v)dG = \int_{\underline{c}}^{\overline{c}} \left[ vq^*_\gamma(c,v) - \underline{v}q^*_\gamma(c,\underline{v}) + t^*_\gamma(c,\underline{v}) - \int_{\underline{v}}^{v} q^*_\gamma(c,x)dx \right] dG$$

If $\gamma > 1/2$, $\int_{\underline{c}}^{\overline{c}} \underline{v}q^*_\gamma(c,\underline{v}) - t^*_\gamma(c,\underline{v})dG = \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{c \leq \underline{v}\}}(\underline{v} - \mathbb{E}_c[c \mid c \leq \underline{v}])dG$ and $\int_{\underline{c}}^{\overline{c}} t^*_\gamma(c,v)dG = G(\psi_{\mathcal{B},\gamma}(v)) \cdot \mu_\gamma(v) = G(v)\mathbb{E}_c[c \mid c \leq v]$. For

$$B(v) := \int_{\underline{c}}^{\overline{c}} \left[ vq^*_\gamma(c,v) - \underline{v}q^*_\gamma(c,\underline{v}) + t^*_\gamma(c,\underline{v}) - \int_{\underline{v}}^{v} q^*_\gamma(c,x)dx \right] dG$$

we have

$$\begin{aligned}
B(v) &= G(v)\mathbb{E}_c[\max\{c, \underline{v}\} \mid c \leq v] - (\underline{v} - \mathbb{E}_c[c \mid c \leq \underline{v}])G(\underline{v}) \\
&= \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{c \leq v\}} \cdot \max\{c, \underline{v}\} - \mathbb{1}_{\{c \leq \underline{v}\}}(\underline{v} - c)dG \\
&= \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{c \leq v\}} c \; dG \\
&= G(v)\mathbb{E}_c[c \mid c \leq v]
\end{aligned}$$

And so, the envelope condition holds over $\gamma > 1/2$ also.

To show $(q^*_\gamma, t^*_\gamma)$ satisfies (S-IR), note that seller trade surplus is (weakly) decreasing in $\gamma$ and is constant over $[1/2, 1]$. As a result, it is sufficient to show $(q^*_{1/2}, t^*_{1/2})$ satisfies (S-IR). We may set $\int_{\underline{c}}^{\overline{c}} \underline{v} q^*_\gamma(c, \underline{v}) - t^*_\gamma(c, \underline{v}) dG = 0$ as a 1/2-maximal outcome. By Fubini, the binding buyer's participation, buyer's incentive compatibility, and Lemma A.1,

$$\begin{aligned}
\int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} t^*_{1/2}(c, v) - q^*_{1/2}(c, v) c dGdF &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{v \geq c\}} \left[\psi_{\mathcal{B},0}(v) - c\right] dGdF \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \frac{G(c)}{g(c)} \mathbb{1}_{\{c \geq v\}} dGdF - \int_{\overline{c}}^{\underline{v}} (1 - F(x)) G(x) dx \\
&= \int_{\underline{c}}^{\max\{\underline{v}, \underline{c}\}} G(x) dx \geq 0
\end{aligned}$$

□

## A.3 Proof of Corollary 2.1

We prove four lemmata. Lemma A.2 derives an expression for expected royalty payments over the Pareto frontier, giving us the bound in point (i). Lemma A.3 shows the royalties, $k^*_\gamma$, are decreasing over $p \in \mu_\gamma([\underline{v}, \overline{v}])$, giving point (ii). Lemma A.4 demonstrates point (iii). Lemma A.5 shows that the interior of $[\underline{c}, \overline{c}] \cap \psi_{\mathcal{B},\gamma}([\underline{v}, \overline{v}])$ is non-empty if and only if $\mu_\gamma([\underline{v}, \overline{v}])$ has strictly positive Lebesgue measure. Therefore, whenever the interior of $[\underline{c}, \overline{c}] \cap \psi_{\mathcal{B},\gamma}([\underline{v}, \overline{v}])$ is non-empty, the buyer posts an interval of prices and from Lemma A.2 these are on average non-positive and from Lemma A.3 are decreasing in prices. Consequently, royalties are necessarily negative for high enough prices, giving point (iv).

**Lemma A.2** *For $\gamma \in [0, 1]$, in the $\gamma$-maximal royalty scheme, $k^*_\gamma$, equilibrium expected royalty payments are non-positive and equal to*

$$K_\gamma = \left[\max\left\{0, \frac{1 - 2\gamma}{1 - \gamma}\right\} - 1\right] \cdot \int_{\underline{v}}^{\overline{v}} G(\psi_{\mathcal{B},\gamma}(x))[1 - F(x)] dx \in [-W^e, 0]$$

*Proof.* Under the $\gamma$-maximal allocation, we mildly abuse notation and write the seller's surplus including their contract payments and their trade surplus as,

$$\begin{aligned}
\pi_\gamma &:= \mathbb{E}_{\omega,v,c,x \sim Bern(a_{k^*_\gamma(\omega,c,p_{k^*_\gamma(v)})})(\omega,c,p_{k^*_\gamma}(v))}[\pi(x, p_{k^*_\gamma}(v), k^*_\gamma; c, \omega)] \\
\hat{\pi}_\gamma &:= \mathbb{E}_{\omega,v,c,x \sim Bern(a_{k^*_\gamma(\omega,c,p_{k^*_\gamma(v)})})(\omega,c,p_{k^*_\gamma}(v))}[\hat{\pi}(x, p_{k^*_\gamma}(v), k^*_\gamma; c, \omega)]
\end{aligned}$$

Then,

$$\begin{aligned}
\pi_\gamma &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{\psi_{\mathcal{B},\gamma}(v) \geq c\}} (p_{k^*_\gamma}(v) - k^*_\gamma(p_{k^*_\gamma}(v)) - c) dGdF \\
&= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{\psi_{\mathcal{B},\gamma}(v) \geq c\}} (\psi_{\mathcal{B},\gamma}(v) - c) dGdF \qquad (\dagger)
\end{aligned}$$

and,

$$\begin{aligned}\hat{\pi}_\gamma &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{\psi_{\mathcal{B},\gamma}(v)\geq c\}}(p_{k_\gamma^*}(v) - c) dG dF \\ &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{\psi_{\mathcal{B},\gamma}(v)\geq c\}}(\mu_\gamma(v) - c) dG dF \end{aligned} \tag{‡}$$

Denote the expected royalty payments in the $\gamma-$maximal outcome as $K_\gamma^*$. We have the following relation

$$\hat{\pi}_\gamma - K_\gamma^* = \pi_\gamma$$

Therefore, by (†) and (‡),

$$\begin{aligned}K_\gamma^* &= \hat{\pi}_\gamma - \pi_\gamma \\ &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{\psi_{\mathcal{B},\gamma}(v)\geq c\}}(\mu_\gamma(v) - \psi_{\mathcal{B},\gamma}(v)) dG dF\end{aligned}$$

Now, using the expression for $\mu_\gamma(v)$ derived in the proof of Theorem 2, $\mu_\gamma(v) = v - \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x))dx$, we obtain by application of Fubini,

$$\begin{aligned}K_\gamma^* &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{\psi_{\mathcal{B},\gamma}(v)\geq c\}}(\mu_\gamma(v) - \psi_{\mathcal{B},\gamma}(v)) dG dF \\ &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{\psi_{\mathcal{B},\gamma}(v)\geq c\}} \begin{pmatrix} -\frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x))dx \\ + \max\left\{0, \frac{1-2\gamma}{1-\gamma}\right\} \cdot \frac{1-F(v)}{f(v)} \end{pmatrix} dG dF \\ &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{\psi_{\mathcal{B},\gamma}(v)\geq c\}} \left( -\frac{1-F(v)}{f(v)} + \max\left\{0, \frac{1-2\gamma}{1-\gamma}\right\} \cdot \frac{1-F(v)}{f(v)} \right) dG dF \\ &= \left[ \max\left\{0, \frac{1-2\gamma}{1-\gamma}\right\} - 1 \right] \cdot \int_{\underline{v}}^{\overline{v}} G(\psi_{\mathcal{B},\gamma}(x))[1-F(x)]dx\end{aligned}$$

Clearly, $K_\gamma^*$ is decreasing in $\gamma$. At $\gamma = 0$, $K_\gamma^* = 0$. At $\gamma = 1$, $\psi_{\mathcal{B},\gamma}(v) = v$ and $\max\left\{0, \frac{1-2\gamma}{1-\gamma}\right\} - 1 = -1$. By Lemma A.1,

$$\begin{aligned}-K_1^* &= \int_{\underline{v}}^{\overline{v}} G(x)[1-F(x)]dx \\ &= \int_{\underline{v}}^{\overline{v}} G(x)[1-F(x)]dx + \int_{\underline{v}}^{\overline{c}} G(x)[1-F(x)]dx - \int_{\underline{v}}^{\overline{c}} G(x)[1-F(x)]dx \\ &= \begin{pmatrix} \int_{\underline{v}}^{\overline{v}} G(x)[1-F(x)]dx + \int_{\underline{v}}^{\overline{c}} G(x)[1-F(x)]dx \\ +W^e - \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \mathbb{1}_{\{v\geq c\}} G(x) dx dF - \int_{\underline{c}}^{\overline{c}} \int_{\underline{v}}^{\overline{v}} \mathbb{1}_{\{v\geq c\}}(1-F(x)) dx dG \end{pmatrix} \\ &= W^e + \begin{bmatrix} \int_{\underline{v}}^{\overline{v}} G(x)[1-F(x)]dx + \int_{\underline{v}}^{\overline{c}} G(x)[1-F(x)]dx \\ - \int_{\underline{c}}^{\overline{c}} G(x)[1-F(x)]dx - \int_{\underline{v}}^{\overline{v}} G(x)[1-F(x)]dx \end{bmatrix} \\ &= W^e\end{aligned}$$

As such, $K_\gamma^* \in [-W^e, 0]$. □

**Lemma A.3** *If $G$ is regular, $k_\gamma^*$ is monotone decreasing over $p \in \mu_\gamma([\underline{v}, \overline{v}])$.*

*Proof.* Over $p \in \mu_\gamma(\psi_{\mathcal{B},\gamma}^{-1}([\underline{c},\overline{c}])) \subseteq \mu_\gamma([\underline{v},\overline{v}])$, royalties are implicitly defined by $k_\gamma^*(p_{k_\gamma^*}(v)) = p_{k_\gamma^*}(v) - \psi_{\mathcal{B},\gamma}(v)$. For $\ell$ a function of bounded variation, denote by $d\ell$ the Lebesgue-Stieltjes (signed) measure induced by $\ell$, i.e., for $a < b$ $d\ell((a,b]) = \ell(b) - \ell(a)$. Note $d\ell$ is well-defined even if $\ell$ is non-differentiable. If $d\ell > (<)0$ then $\ell$ is increasing (decreasing). Because $F$ has increasing hazard rate, $p_{k_\gamma^*}$, $\psi_{\mathcal{B},\gamma}$, and $\psi_{\mathcal{B},\gamma}^{-1}$ are monotone functions on compact intervals and are therefore bounded. Hence, the Lebesgue-Stieltjes measure is well-defined for these functions.

We first prove that $d(\psi_{\mathcal{B},\gamma}^{-1}) \leq \lambda$ where $\lambda$ is the Lebesgue measure. Because $F$ has increasing hazard rate, fix any $a < b$, then

$$\psi_{\mathcal{B},\gamma}(b) - \psi_{\mathcal{B},\gamma}(a) = b - a - \underbrace{\max\left\{0, \frac{2\gamma-1}{1-\gamma}\right\}}_{\geq 0} \underbrace{\left(\frac{1-F(b)}{f(b)} - \frac{1-F(a)}{f(a)}\right)}_{\leq 0 \text{ as } F \text{ regular}} \geq b - a \quad (\sim)$$

For any $a < b$ in the range of $\psi_{\mathcal{B},\gamma}^{-1}$, because the inverse in increasing, there exists $x < y$ with $\psi_{\mathcal{B},\gamma}^{-1}(x) = a < b = \psi_{\mathcal{B},\gamma}^{-1}(y)$. Plugging into $(\sim)$, $d(\psi_{\mathcal{B},\gamma}^{-1})((x,y]) = \psi_{\mathcal{B},\gamma}^{-1}(y) - \psi_{\mathcal{B},\gamma}^{-1}(x) \leq y - x$. Therefore, $\psi_{\mathcal{B},\gamma}^{-1}$ has Lipschitz constant 1 and we have $\frac{d(\psi_{\mathcal{B},\gamma}^{-1})}{d\lambda} \leq 1$. As a result, for any non-negative integrable function $\ell$, $\int \ell d(\psi_{\mathcal{B},\gamma}^{-1}) \leq \int \ell d\lambda$.

Because $k_\gamma^*(p_{k_\gamma^*}(v)) = p_{k_\gamma^*}(v) - \psi_{\mathcal{B},\gamma}(v)$, we have the measure identity, $d(k_\gamma^* \circ p_{k_\gamma^*}) = dp_{k_\gamma^*} - d\psi_{\mathcal{B},\gamma}$. Fix $v_1, v_2 \in [\underline{v},\overline{v}]$ with $v_1 < v_2$ offering prices $p_{k_\gamma^*}(v_1), p_{k_\gamma^*}(v_1) \in \mu_\gamma([\underline{v},\overline{v}])$. We have, $\underline{c} < \psi_{\mathcal{B},\gamma}(v_1) \leq \psi_{\mathcal{B},\gamma}(v_1)$. Then,

$$\begin{aligned}
d(k_\gamma^* \circ p_{k_\gamma^*})((v_1,v_2]) &= dp_{k_\gamma^*}((v_1,v_2]) - d\psi_{\mathcal{B},\gamma}((v_1,v_2]) \\
&= \begin{bmatrix} v_2 - v_1 - d\left[\frac{1}{G(\psi_{\mathcal{B},\gamma}(\cdot))}\int_{\underline{v}}^{\cdot} G(\psi_{\mathcal{B},\gamma}(x))dx\right]((v_1,v_2]) \\ -d\psi_{\mathcal{B},\gamma}((v_1,v_2]) \end{bmatrix} \\
&= \int_{v_1}^{v_2} \left[\underbrace{\frac{g(\psi_{\mathcal{B},\gamma}(v))}{G^2(\psi_{\mathcal{B},\gamma}(v))}\int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x))dx}_{:=B(v)} - 1\right] d\psi_{\mathcal{B},\gamma}(v)
\end{aligned}$$

Because $\psi_{\mathcal{B},\gamma}$ and $G/g$ are both increasing,

$$\begin{aligned}
B(v) &= \frac{g(\psi_{\mathcal{B},\gamma}(v))}{G^2(\psi_{\mathcal{B},\gamma}(v))}\int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x))dx \\
&= \frac{g(\psi_{\mathcal{B},\gamma}(v))}{G^2(\psi_{\mathcal{B},\gamma}(v))}\int_{\underline{v}}^{v} \frac{G(\psi_{\mathcal{B},\gamma}(x))}{g(\psi_{\mathcal{B},\gamma}(x))} g(\psi_{\mathcal{B},\gamma}(x))dx \\
&< \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))}\int_{\underline{v}}^{v} g(\psi_{\mathcal{B},\gamma}(x))dx \\
&= \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))}\int_{\psi_{\mathcal{B},\gamma}(\underline{v})}^{\psi_{\mathcal{B},\gamma}(v)} g(u)d(\psi_{\mathcal{B},\gamma}^{-1}(u)) \\
&\leq \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))}\int_{\psi_{\mathcal{B},\gamma}(\underline{v})}^{\psi_{\mathcal{B},\gamma}(v)} g(u)du \\
&= \frac{G(\psi_{\mathcal{B},\gamma}(v)) - G(\psi_{\mathcal{B},\gamma}(\underline{v}))}{G(\psi_{\mathcal{B},\gamma}(v))}
\end{aligned}$$

As a result, $B(v) < 1$. As $d\psi_{\mathcal{B},\gamma} > 0$,

$$d(k^*_\gamma \circ p_{k^*_\gamma})((v_1, v_2]) = \int_{v_1}^{v_2} (B(v) - 1) d\psi_{\mathcal{B},\gamma}(v) < 0$$

Because $dp_{k^*_\gamma} > 0$, this implies $dk^*_\gamma < 0$ and hence that $k^*_\gamma$ is strictly decreasing over $p \in \mu_\gamma([\underline{v}, \overline{v}])$. $\square$

**Lemma A.4** *For* $1/2 > \gamma > \gamma'$ *and* $p \in \mu_\gamma([\underline{v}, \overline{v}]) \cap \mu_{\gamma'}([\underline{v}, \overline{v}])$, $k^*_\gamma(p) < k^*_{\gamma'}(p)$.

*Proof.* As in Lemma A.3, we note royalties are implicitly defined by $k^*_\gamma(p_{k^*_\gamma}(v)) = p_{k^*_\gamma}(v) - \psi_{\mathcal{B},\gamma}(v)$. Because $\psi_{\mathcal{B},\gamma}(v)$ for fixed $v$ is strictly increasing in $\gamma$,

$$p_{k^*_\gamma}(v) = \mu_\gamma(v) = \begin{cases} \mathbb{E}_c[\psi^{-1}_{\mathcal{B},\gamma}(c) \mid c \le \psi_{\mathcal{B},\gamma}(v)] & \text{if } \gamma \le 1/2, \\ \mathbb{E}_c[c \mid c \le v] & \text{if } \gamma > 1/2 \end{cases}$$

is increasing $\gamma$ for $\gamma < 1/2$. Therefore, since $\frac{d}{d\gamma}(k^*_\gamma \circ p_{k^*_\gamma})(v) = \frac{dk^*_\gamma(p_{k^*_\gamma}(v))}{d\gamma} \frac{dp_{k^*_\gamma}(v)}{d\gamma}$ if $\frac{d}{d\gamma} k^*_\gamma(p_{k^*_\gamma}(v)) < 0$ then $\frac{dk^*_\gamma(p_{k^*_\gamma}(v))}{d\gamma} < 0$. Note that any value type $v$ posting $p \in \mu_\gamma([\underline{v}, \overline{v}]) \cap \mu_{\gamma'}([\underline{v}, \overline{v}])$ must satisfy $\psi_{\mathcal{B},\tilde{\gamma}}(v) > \underline{c}$ for all $\tilde{\gamma} \in [\gamma', \gamma]$. Consider for $\gamma < 1/2$,

$$\begin{aligned}
\frac{d}{d\gamma} k^*_\gamma(p_{k^*_\gamma}(v)) &= \frac{d\mu_\gamma(v)}{d\gamma} - \frac{d\psi_{\mathcal{B},\gamma}(v)}{d\gamma} \\
&= \frac{d}{d\gamma}\left(v - \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x)) dx\right) - \frac{1}{(1-\gamma)^2} \frac{1 - F(v)}{f(v)} \\
&= \begin{pmatrix} \frac{1}{(1-\gamma)^2} \frac{1-F(v)}{f(v)} \left[\frac{g(\psi_{\mathcal{B},\gamma}(v))}{G(\psi_{\mathcal{B},\gamma}(v))^2} \int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x)) dx\right] \\ - \frac{1}{(1-\gamma)^2} \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \int_{\underline{v}}^{v} g(\psi_{\mathcal{B},\gamma}(x)) \frac{1-F(x)}{f(x)} dx - \frac{1}{(1-\gamma)^2} \frac{1-F(v)}{f(v)} \end{pmatrix} \\
&\le \frac{1}{(1-\gamma)^2} \frac{1 - F(v)}{f(v)} \begin{bmatrix} \frac{g(\psi_{\mathcal{B},\gamma}(v))}{G(\psi_{\mathcal{B},\gamma}(v))^2} \int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x)) dx \\ - \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \int_{\underline{v}}^{v} g(\psi_{\mathcal{B},\gamma}(x)) dx - 1 \end{bmatrix} \\
&= \frac{1}{(1-\gamma)^2} \frac{1 - F(v)}{f(v)} \begin{bmatrix} \frac{g(\psi_{\mathcal{B},\gamma}(v))}{G(\psi_{\mathcal{B},\gamma}(v))^2} \int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x)) dx \\ - \frac{1}{G(\psi_{\mathcal{B},\gamma}(v))} \int_{\underline{v}}^{v} \frac{g(\psi_{\mathcal{B},\gamma}(x))}{G(\psi_{\mathcal{B},\gamma}(x))} G(\psi_{\mathcal{B},\gamma}(x)) dx - 1 \end{bmatrix} \\
&\le \frac{1}{(1-\gamma)^2} \frac{1 - F(v)}{f(v)} \begin{bmatrix} \frac{g(\psi_{\mathcal{B},\gamma}(v))}{G(\psi_{\mathcal{B},\gamma}(v))^2} \int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x)) dx \\ - \frac{g(\psi_{\mathcal{B},\gamma}(v))}{G(\psi_{\mathcal{B},\gamma}(v))^2} \int_{\underline{v}}^{v} G(\psi_{\mathcal{B},\gamma}(x)) dx - 1 \end{bmatrix} \\
&= -\frac{1}{(1-\gamma)^2} \frac{1 - F(v)}{f(v)} < 0
\end{aligned}$$

Therefore, for $1/2 > \gamma > \gamma'$ and $p \in \mu_\gamma([\underline{v}, \overline{v}]) \cap \mu_{\gamma'}([\underline{v}, \overline{v}])$, $k^*_\gamma(p) < k^*_{\gamma'}(p)$. $\square$

**Lemma A.5** *The interior of* $[\underline{c}, \overline{c}] \cap \psi_{\mathcal{B},\gamma}([\underline{v}, \overline{v}])$ *is non-empty if and only if* $\mu_\gamma([\underline{v}, \overline{v}])$ *has strictly positive Lebesgue measure.*

*Proof.* Recall $\mu_\gamma(v) = \mathbb{E}_c[\psi^{-1}_{\mathcal{B},\gamma}(c) \mid c \le \psi_{\mathcal{B},\gamma}(v)]$. Then,

$$\begin{aligned}
\mu_\gamma(\underline{v}) &= \underline{v} \\
\mu_\gamma(\overline{v}) &= \mathbb{E}_c[\psi^{-1}_{\mathcal{B},\gamma}(c) \mid c \le \psi_{\mathcal{B},\gamma}(\overline{v})] \\
&= \overline{v} - \frac{1}{G(\psi_{\mathcal{B},\gamma}(\overline{v}))} \int_{\underline{v}}^{\overline{v}} G(\psi_{\mathcal{B},\gamma}(x)) dx
\end{aligned}$$

We first prove that if $[\underline{c},\overline{c}]\cap\psi_{\mathcal{B},\gamma}([\underline{v},\overline{v}])$ has non-empty interior, $\mu_\gamma([\underline{v},\overline{v}])$ has strictly positive Lebesgue measure. If the interior is non-empty then either $\overline{v}>\psi^{-1}_{\mathcal{B},\gamma}(\underline{c})>\underline{v}$ or $\overline{v}>\psi^{-1}_{\mathcal{B},\gamma}(\overline{c})>\underline{v}$.

If $\overline{v}>\psi^{-1}_{\mathcal{B},\gamma}(\overline{c})>\underline{v}$, $G(\psi_{\mathcal{B},\gamma}(\overline{v}))=1$ but $\int_{\underline{v}}^{\overline{v}}G(\psi_{\mathcal{B},\gamma}(x))dx<\overline{v}-\underline{v}$, so $\mu_\gamma(\overline{v})>\underline{v}=\mu_\gamma(\underline{v})$. Because $\mu_\gamma$ is continuous, by intermediate value theorem $\mu_\gamma([\underline{v},\overline{v}])=[\mu_\gamma(\underline{v}),\mu_\gamma(\overline{v})]$ and $\mu_\gamma(\overline{v})-\mu_\gamma(\underline{v})>0$.

If $\overline{v}>\psi^{-1}_{\mathcal{B},\gamma}(\underline{c})>\underline{v}$, $\frac{1}{G(\psi_{\mathcal{B},\gamma}(\overline{v}))}\geq 1$ and $\int_{\underline{v}}^{\overline{v}}G(\psi_{\mathcal{B},\gamma}(x))dx<\overline{v}-\underline{v}$, and similar logic applies as in the other case.

Second, that if $\mu_\gamma([\underline{v},\overline{v}])$ has strictly positive Lebesgue measure then the interior of $[\underline{c},\overline{c}]\cap\psi_{\mathcal{B},\gamma}([\underline{v},\overline{v}])$ is non-empty follows immediately from the definition of $\mu_\gamma$. □

## A.4 Proof of Proposition 2

From Proposition 9 of Yang and Yang (2025), any MS-implementable interim payoff may be achieved through randomisation over markup-pooling mechanisms. For any mechanism $(q,t)$ which arises from a randomisation over markup-pooling mechanisms, we find a system of transfers $\tilde{t}$ such that $(q,\tilde{t})$ is contract implementable and such that $(q,t)$ and $(q,\tilde{t})$ are ex ante payoff equivalent.

From Yang and Yang (2025) for any markup-pooling outcome $(q,t)$ the allocation rule may be written as

$$q(c,v)=\begin{cases}\mathbb{1}\{v\geq\xi(c)\} & c\notin[c_L,c_H]\\ \lambda\mathbb{1}\{v\geq\xi(c_L)\}+(1-\lambda)\mathbb{1}\{v\geq\xi(c_H)\} & c\in[c_L,c_H]\end{cases}$$

for some non-decreasing function $\xi$, interval $[c_L,c_H]$, and constant $\lambda\in[0,1]$.

It is routine to check any such $q(c,v)$ is non-increasing in $c$ for each fixed $v$ and non-decreasing in $v$ for each fixed $c$. Further, since mixtures preserve monotonicity, randomisations over markup-pooling mechanisms are also non-increasing in $c$ for each $v$ and non-decreasing in $v$ for each $c$.

*Proof.* Let $(q,t)$ be a randomisation over markup-pooling mechanisms which is MS-implementable. Define $\tilde{t}(c,v)=d(v)q(c,v)$ where

$$d(v)\int_{\underline{c}}^{\overline{c}}q(c,v)dG=\int_{\underline{c}}^{\overline{c}}\left[vq(c,v)-\underline{v}q(c,\underline{v})+t(c,\underline{v})-\int_{\underline{v}}^{v}q(c,x)dx\right]dG \qquad (\diamond)$$

and $d(v)=\min\{\underline{c},\underline{v}\}$ for all $v$ such that $0=\int_{\underline{c}}^{\overline{c}}q(c,v)dG$. Since $(q,t)$ is MS-implementable, the buyer's envelope condition implies that by construction of $\tilde{t}$, $\int_{\underline{c}}^{\overline{c}}t(c,v)dG=\int_{\underline{c}}^{\overline{c}}\tilde{t}(c,v)dG$.

Because $(q,t)$ is MS-implementable $0\leq\int_{\underline{c}}^{\overline{c}}\underline{v}q(c,\underline{v})-t(c,\underline{v})dG$, giving $d(v)\leq v$ for all $v$.

Suppose $v'>v$ and $d(v')=d(v):=d$. We show this implies $\int_{\underline{c}}^{\overline{c}}q(c,v')dG=\int_{\underline{c}}^{\overline{c}}q(c,v)dG$. As $\int_{\underline{c}}^{\overline{c}}q(c,v)dG$ is non-decreasing in $v$, $\int_{\underline{c}}^{\overline{c}}q(c,v')dG\geq\int_{\underline{c}}^{\overline{c}}q(c,v)dG$. Suppose, seeking contradiction that $\int_{\underline{c}}^{\overline{c}}q(c,v')dG>\int_{\underline{c}}^{\overline{c}}q(c,v)dG$.

By subtracting (◇) at $v'$ and $v$,

$$d\left(\int_{\underline{c}}^{\overline{c}} q(c,v')dG - \int_{\underline{c}}^{\overline{c}} q(c,v)dG\right) = \begin{bmatrix} v'\left(\int_{\underline{c}}^{\overline{c}} q(c,v')dG\right) \\ -v\left(\int_{\underline{c}}^{\overline{c}} q(c,v)dG\right) - \int_v^{v'}\int_{\underline{c}}^{\overline{c}} q(c,x)dGdx \end{bmatrix}$$

Consequently, we have the two conditions

$$(v'-d)\left(\int_{\underline{c}}^{\overline{c}} q(c,v')dG - \int_{\underline{c}}^{\overline{c}} q(c,v)dG\right) = \int_v^{v'}\left(\int_{\underline{c}}^{\overline{c}} q(c,x)dG - \int_{\underline{c}}^{\overline{c}} q(c,v)dG\right)dx$$
$$(d-v)\left(\int_{\underline{c}}^{\overline{c}} q(c,v')dG - \int_{\underline{c}}^{\overline{c}} q(c,v)dG\right) = \int_v^{v'}\left(\int_{\underline{c}}^{\overline{c}} q(c,v')dG - \int_{\underline{c}}^{\overline{c}} q(c,x)dG\right)dx$$

Because $\int_{\underline{c}}^{\overline{c}} q(c,v')dG > \int_{\underline{c}}^{\overline{c}} q(c,v)dG$, the last two equations imply $v < d < v'$. Yet this implies $v < d(v)$, a contradiction.

Hence, $d(v) = d(v')$ implies $\int_{\underline{c}}^{\overline{c}} q(c,v')dG = \int_{\underline{c}}^{\overline{c}} q(c,v)dG$. Now, if $v' > v$ and $d(v) = d(v')$,

$$0 = \int_{\underline{c}}^{\overline{c}} q(c,v') - q(c,v)dG$$

Since $q(c,v)$ is non-decreasing in $v$ for each $c$ this implies $q(c,v') - q(c,v) \geq 0$ for all $c$ and thus that $q(c,v') = q(c,v)$ for all $c$ — up to $G$-null sets. Whence, $q$ factors through $d$, as does $\tilde{t}$ since $\tilde{t}(c,v) = d(v)q(c,v)$.

By construction, $(q,\tilde{t})$ satisfies the conditions of Lemma 1 and so is contract implementable. Further, as the allocation rule is the same in $(q,t)$ and $(q,\tilde{t})$ and because

$$\int_{\underline{c}}^{\overline{c}} t(c,v)dG = \int_{\underline{c}}^{\overline{c}} \tilde{t}(c,v)dG \implies \int_{\underline{v}}^{\overline{v}}\int_{\underline{c}}^{\overline{c}} t(c,v)dGdF = \int_{\underline{v}}^{\overline{v}}\int_{\underline{c}}^{\overline{c}} \tilde{t}(c,v)dGdF$$

ex ante payoffs are the same under the two outcomes. The result follows. □

# B Computation of UPS

We have $c \sim U[0,1]$, $v \sim U[0,1]$. First, in any buyer incentive compatible mechanism, by payoff equivalence and Fubini, the buyer and seller surpluses are equal to

$$BS(q) = \int_0^1\int_0^1 q(c,v)[1-v]dvdc$$
$$SS(q) = \int_0^1\int_0^1 q(c,v)[2v-1-c]dvdc$$

We derive the contract-implementable frontier. From Theorem 2, the frontier is spanned by $q^*_\gamma(c,v) = \mathbb{1}\{\psi_{\mathcal{B},\gamma}(v) \geq c\}$. Define $\Gamma = \max\{0, (1-2\gamma)/(1-\gamma)\}$. Then, the contract-implementable utility possibility frontier is defined by

$$\mathbb{U}_C = \left\{(U_B, U_S) = \left(\frac{1}{6(1+\Gamma)^2}, \frac{\Gamma}{3(1+\Gamma)^2}\right) \middle| \gamma \in [0,1]\right\}$$

This gives the blue boundary for $U_B \geq 1/24$.

For the boundary when $U_B \in [0, 1/24)$, we solve the problem

$$V(\overline{u}) = \max_q SS(q)$$

$$st. \begin{cases} \int_0^1 \int_0^1 q(c,v)[2v-1-c]dvdc \geq 0 \\ \overline{u} = \int_0^1 \int_0^1 q(c,v)[1-v]dvdc \end{cases}$$

for $\overline{u} \in [0, 1/24)$. This solves to $V(\overline{u}) = \frac{\sqrt{6\overline{u}}}{3} - 2\overline{u}$. This defines the contract feasible set as the origin, $(U_B, U_S) = (0,0)$ is feasible and the utility possibility set is convex.

For the Myerson-Satterthwaite frontier, we employ the techniques of Ledyard and Palfrey (1999) and Williams (1987). Using Theorem 1 of Myerson and Satterthwaite (1983), we solve the program

$$\max_q \gamma BS(q) + (1-\gamma)SS(q)$$

$$st. \int_0^1 \int_0^1 q(c,v)\,[2v-1-2c]\,dvdc \geq 0$$ [15]

Consider the Lagrangian,

$$\mathcal{L} = \gamma BS(q) + (1-\gamma)SS(q) + \lambda \int_0^1 \int_0^1 (2v-1-2c)dcdv = \int_0^1 \int_0^1 q(c,v)\phi_{\gamma,\lambda}(c,v)dcdv$$

where $\phi_{\gamma,\lambda}(c,v) := \gamma(1-v)+(1-\gamma)(2v-1-c)+\lambda(2v-1-2c)$. The optimal allocation is then $q_{\gamma,\lambda}(c,v) = \mathbb{1}\{\phi_{\gamma,\lambda}(c,v) \geq 0\}$. The problem becomes selecting $\lambda_\gamma = \lambda$ such that the ex ante budget constraint is satisfied and, if binding, satisfied with equality. One may verify that if $\lambda_\gamma = 0$, $q_{\gamma,\lambda_\gamma}$ violates the budget constraint for all $\gamma > 0$. At $\gamma = 0$, the unconstrained maximum is achieved by $q_{0,0}(c,v) = \mathbb{1}\{v \geq (1+c)/2\}$.

For $\gamma > 0$, $\lambda_\gamma > 0$ is chosen to satisfy

$$\int_0^1 \int_0^1 q_{\gamma,\lambda_\gamma}(c,v)[2v-1-2c]dcdv = 0$$

Given the formula for $q_{\gamma,\lambda_\gamma}$, the equation reduces to a quadratic in $\lambda_\gamma$. Selecting the postiive root, we find $\lambda_\gamma = \frac{2\gamma-1+\sqrt{3\gamma^2-3\gamma+1}}{2}$ note that $\lambda_0 = 0$ so encompasses the edge case. We may then derive buyer and seller surpluses over the frontier in closed form. Then,

$$BS(q^{MS}_{\gamma,\lambda_\gamma}) = \frac{(\sqrt{3\gamma^2-3\gamma+1}+1)^3}{48(\gamma+\sqrt{3\gamma^2-3\gamma+1})(1-\gamma+\sqrt{3\gamma^2-3\gamma+1})^2}$$
$$SS(q^{MS}_{\gamma,\lambda_\gamma}) = \frac{(\sqrt{3\gamma^2-3\gamma+1}+1)^3}{48(\gamma+\sqrt{3\gamma^2-3\gamma+1})^2(1-\gamma+\sqrt{3\gamma^2-3\gamma+1})}$$

so the MS-ex ante frontier is

$$\mathbb{U}_{MS} := \{(U_B, U_S) = (BS(q^{MS}_{\gamma,\lambda_\gamma}), SS(q^{MS}_{\gamma,\lambda_\gamma})) \mid \gamma \in [0,1]\}$$

[15]Strictly speaking, this is the relaxed problem absent the monotonicity constraints implied by incentive compatibility. Nonetheless, because cost and value distributions are regular (uniform), the solution to this relaxed problem naturally satisfies these additional constraints.

To get the left-hand side of the MS 'teardrop', we solve the program

$$W(\overline{u}) = \max_q SS(q)$$

$$st. \begin{cases} \int_0^1 \int_0^1 q(c,v)[2v-1-2c]dvdc \geq 0 \\ \overline{u} = \int_0^1 \int_0^1 q(c,v)[1-v]dvdc \end{cases}$$

For $\overline{u} \in [0, 1/24)$ This has solution $W(\overline{u}) = V(\overline{u}) = \frac{\sqrt{6\overline{u}}}{3} - 2\overline{u}$.

In Myerson-Satterthwaite, buyer and seller are symmetric in the mechanism. Therefore, since cost and value distributions are also symmetric in the uniform case we are considering, we can trace out the right-hand side of the MS 'teardrop' by mapping out the graph $(W(\underline{u}), \underline{u})$ for $\underline{u} \in [0, 1/24)$, i.e., by reflecting the graph about the line $U_B = U_S$.

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