Vertical Mergers in Ecosystems with Consumer Hold-up*

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November 5, 2023

Abstract

An ecosystem comprises all downstream products that employ a certain upstream input. In many cases, final consumers make irreversible investments to join an ecosystem before downstream prices are set. By committing to buy products that use the specific ecosystem input, they are at risk of being held-up. Unable to observe future prices, consumers base their decisions on what they observe about the market structure within each ecosystem, including vertical contracts signed by the upstream firms. By entering into vertical agreements with multiple competing downstream firms, thus creating a credible expectation of lower prices, an upstream firm is able to mitigate consumers' hold-up problem and, as a result, increase ecosystem demand. Our main observation is that, in contrast to conventional wisdom, an upstream monopolist merging with one of its downstream affiliates will find it profitable to continue to serve downstream competitors, even when products sold downstream are homogeneous.

*We thank Rodrigo Montes, Dan O'Brien, Guillaume Duquesne, Salvatore Piccolo, Shiva Shekhar, the editor and two anonymous referees for comments. Joe Basford provided valuable research assistance. The first two authors gratefully acknowledge funding from Nvidia Corporation.

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1 Introduction

A substantial body of work has emphasized that vertical mergers involving an upstream monopolist may result in complete *input* foreclosure of downstream firms, especially of those producing close substitutes of the integrated firm's final product (e.g., see Rey & Tirole (2007)).[1] In this paper we illustrate how this conclusion is overturned in the presence of consumer hold-up and ecosystem competition, or, as we shall say, with "ecosystem effects".

We informally define an "ecosystem" as a supply-chain for the production of several final products. An ecosystem includes an upstream product (e.g., an operating system) and all downstream products that rely on it as an input or as an an essential complement with which they inter-operate (e.g., software applications). In our analysis, what distinguishes ecosystems from traditional supply chains is the potential for consumer *hold-up*.[2] That is, final clients decide, before prices of final products are set and for the long-term, which ecosystem to join. Being unable to observe future prices, consumers are at risk of being held-up and rely on signals about their future consumer surplus, such as downstream market structure, when deciding whether to patronize a certain ecosystem. Then, we speak of "ecosystem effects" when the *volume* of customers joining the ecosystem, i.e., committing to purchasing one of the ecosystem's downstream products (or otherwise incur switching costs) depends on *expected* consumer surplus, which in turn depends on the within-ecosystem *market structure* and, possibly, on the market structure of competing ecosystems.

This papers focuses on the effects that a merger between an upstream monopolist who controls access to the ecosystem and one of its downstream affiliates (i.e., a downstream firm within the ecosystem) has on input supply and within-ecosystem competition. For example, consider a merger between the owner of an architecture and one of the chip-makers that produce chips based on such architecture, a case which we will discuss in length in the concluding section. As another example, consider an OS developer merging with an application developer, or a payment card scheme merging with an acquiring bank.

Our main observation is the following. When ecosystem effects are present, the integrated upstream firm may find it beneficial to sign (non-foreclosing) contracts with downstream competitors, even with those selling close-substitutes. In fact, unless the integrated entity

[1]If upstream monopolist is not able to extract the entire downstream surplus, the integrated firm benefits from foreclosure in two ways. First, it shifts profits away from competitors toward the integrated firm. Second, it eliminates downstream competition, facilitating extraction of downstream monopoly profits. Despite exclusion, integration often benefits consumers because, unless the downstream market is competitive or the upstream/downstream negotiation is frictionless, it eliminates double marginalization (see Tirole (1988)).

[2]An economic definition of ecosystem can be traced back to Moore (1993): "companies coevolve capabilities around a new innovation: they work cooperatively and competitively to support new products, satisfy customer needs, and eventually incorporate the next round of innovations." While the emphasis of Moore (1993) is on biological analogies, ours is on the strategic interlink of companies within the same supply-chains.

can sign long-term contracts with its final consumers (which is often impractical as the downstream product might be still in development), it is unable to otherwise credibly promise low downstream prices in order to attract them into its ecosystem. Then, preserving within-ecosystem competition accomplishes the goal of credibly committing the integrated firm to keep within-ecosystem (downstream) prices low. This attracts consumers into the ecosystem, typically benefiting the upstream firm more than foreclosing downstream rivals.

These ideas may contribute to explain why vertical integration is often not followed by input foreclosure in industries that form business ecosystems, where consumers at various levels of the supply chain sustain start-up costs, even in cases where standard theory would unambiguously predict otherwise (e.g., when downstream products enjoy high margins and are not sufficiently differentiated). For instance, when Google started producing mobile phones in competition with Apple, who was running a tightly closed ecosystem, it did not foreclose access to the Android operating system to competing phone producers. This choice may have helped Google to grow the Android ecosystem, attracting consumers and software developers in a way that would have not been possible had Google attempted to extract rents by monopolizing the downstream Android phone market with its own smart-phones.

We formalize our argument within the following model. Two upstream firms, who control two different ecosystems, move first and make take-it-or-leave-it offers, each to their own downstream producers. Having observed the contracts between upstream and downstream producers consumers decide whether to join one ecosystem or the other.[3] Following these decisions, consumer valuations for the downstream product are realized and downstream competition takes place. We stack the deck in favor of post-merger foreclosure by assuming that downstream producers sell a homogeneous product and that the interplay of upstream pricing and downstream competition does not fully dissipate downstream profits. Therefore, we treat the case where downstream profit can be extracted frictionlessly as a limit case (i.e., we allow the upstream firm to charge two-part tariffs, but not to extract the entire downstream profit through the fixed fee) and we assume competition takes place a' la Cournot, a scenario that results in positive profits for downstream firms.

Within the model above, we obtain one general insight. When a market presents ecosystem effects, competing downstream firms will not be fully foreclosed post-merger. If anything, input foreclosure will be partial. Post-merger, the main economic trade-off for the upstream firm is one between achieving a *higher degree of downstream surplus appropriation* by raising the final price through higher wholesale prices, versus *higher volume*, which is achieved by means of a commitment to lower final prices, which in turn, requires serving competing firms

[3]Observability of vertical contracts is crucial but need not apply literally. In practice, consumers must receive a signal of the competitiveness of the downstream market. For instance, which firms are affiliated to an ecosystem via long-term contracts (or licensing) and are developing products for its final market will usually be known to final consumers before they start making irreversible ecosystem-specific investments.

(i.e., the lower price due to elimination of double marginalization is not sufficient). Under standard assumptions, we show that, at the margin, reducing the wholesale price below the fully-foreclosing wholesale price, which is equal to the downstream monopoly price, always benefits the integrated firm.

To obtain sharper results and a closed-form characterization, we focus on a case where one of the ecosystem is closed and operates as an integrated single firm and both the distribution of consumer valuations and the demand to join the ecosystem given expected prices are linear. We are then able to perform comprehensive comparative statics. Beyond the general result that there is no full foreclosure, we obtain the following additional results. First, we show that, as one would expect, wholesale prices are decreasing in the intensity of ecosystem effects (i.e., the degree of product homogeneity between the two competing ecosystems), and are increasing in the level of frictions in upstream/downstream pricing (i.e., the share of downstream profit that can be extracted via fixed fees). This implies an analogous negative relationship between intensity of ecosystem effects and within-ecosystem final price.

Second, we show that as the strength of ecosystem effects increases, the profit of downstream competitors converges to the pre-merger level, and so does the joint profit of the merging firms. Therefore, with strong ecosystem effects, we observe that the vertical merger may have little strategic impact on the profitability of the firms within the ecosystem. Welfare effects are shaped mainly by ecosystem effects, e.g., the intensity of between-ecosystem competition.[4] Intuitively, due to the importance of ecosystem effects, the merged firm will not have an interest in disadvantaging the non-merged downstream competitors, as this would severely affect the volume of consumers joining the ecosystem.

Finally, the benefits of the merger to the parties involved depend on the values of parameters. If, pre-merger, the ability to extract downstream surplus is below a certain middle-level threshold (e.g., at the extreme, if the upstream firm uses linear wholesale prices), then both the merged firm and consumers benefit from the merger. Following the merger, the upstream firm has more at stake in raising volume and therefore engages in more aggressive ecosystem competition for consumers. The incentives to grow the ecosystem are stronger and, in addition, double-marginalisation with one of the firms is eliminated. Downstream competitors also benefit from the merger if ecosystem effects are strong.

On the other hand, when a large share of downstream profit can be extracted via fixed fees without raising prices, whether the merger is beneficial for consumers and the merging firms depends on the strength of ecosystem effects. Somewhat surprisingly, very strong ecosystem effects (i.e., intense ecosystem competition) may make the merger unprofitable and harm

[4]Vertical integration is unlikely to be motivated by a desire to reestablish market power, when that power to raise prices is absent to begin with, due to strong inter-brand competition. With ecosystem effects due to hold-up problem of final consumers, strong inter-brand competition also results in unwillingness of the vertically integrated firm to extract intra-brand profits of downstream competitors.

consumers. The intuition comes from the observation that the ability to impose non-linear prices implies the upstream firm has a good control of downstream behavior and sets very low unit wholesale prices, often below cost. But then, if this is the case, the merger may imply a binding loss of commitment to keeping prices low, as, effectively, the internal transfer-price must now raise to zero. This loss, which is greater when ecosystem effects are stronger, can be partially, but not fully, compensated post-merger by reducing even further the wholesale prices charged to the downstream competitors. In this case, while the merging firms do not jointly benefit from integration, nor normally do consumers, downstream competitors profit.

The paper contributes to the extensive literature on vertical control by highlighting a channel through which the provider of an essential input may *not* benefit from foreclosing downstream competition, even when doing so would allow to restore monopoly power in the downstream market. In particular, committing not to foreclose solves consumers hold-up, when the choice of ecosystem is ex-ante and there are switching costs. A somewhat related idea has been developed in Rey & Tirole (2007, Appendix A), although in their work it's downstream firms that suffer from a hold-problem. In particular, they sketch a model whereby downstream firms must sink investments that orient their technology toward one or some other upstream firm. They then choose the upstream firm which guarantees them the highest profit. Because downstream firms face the risk of being foreclosed when dealing with an integrated upstream firm, the upstream firm may benefit from not integrating even if this results in its inability to enforce a monopoly outcome downstream. In a similar vein, but conversely, Chemla (2003) argues that downstream competition may protect the investment of the upstream firm against expropriation, when downstream firms have bargaining power. A crucial assumption in his model is that the upstream firm has a convex cost function, which implies that it makes a profit when bargaining with two or more firms having bargaining power and making take-it-or-leave-it offers, but not when it bargains with one.

The related idea that, with inter-brand competition, vertical separation can be used as a commitment device to *raise* prices has been explored in Bonanno & Vickers (1988), building on the seminal contribution on delegation by Vickers (1985). In their paper, upstream firms prefer to sell via single independent retailers. Separation confers the upstream firms with the ability to commit to raise downstream costs, which triggers a price increase that benefits every upstream firm when competition is in price, because prices are strategic complements (see Bulow et al. (1985)). When competition is in quantity, delegation is preferable to control from an individual-firm perspective, but in equilibrium pushes firms toward lower prices, because of strategic substitutability. However, in contrast to what happens in our model, suppressing intra-brand competition is not harmful to the ability of upstream firms to compete. In fact, as Rey & Stiglitz (1995) show, vertical restraints that establish a downstream monopoly, for instance exclusive territories or by excluding all but one of the downstream firms, can also be used in conjunction with commitment to soften inter-brand competition. We reach a different conclusion because competition is for-the-market and committing to low prices is only possible by affecting the downstream market structure.

We consider a setting where, by affecting consumer expectations, the presence of multiple downstream firms increases demand.[5] In practice, it is well documented that the possibility of second-sourcing makes products more attractive, mitigating both the risk of supplier-failure and hold-up. For instance, Li & Debo (2009) indicate that Apple normally chooses to acquire its inputs from at least two suppliers. Many other firms in the semiconductor industry have historically adopted a similar strategy (e.g., see Taylor (1984), Sirbu and Hughes (1986) and several issues of *Electronic News*). Explanations of second sourcing which are based on start-up costs and are closely related to ours have been previously advanced in the literature. In Shepard (1987) one monopolistic firm is able to contract on prices but not on quality. If consumers face start-up costs, then licensing its technology to a second firm is a credible commitment to provide high-quality and often raises total industry profit. In Farrell & Gallini (1988) a monopolist may find it in its interest to commit to open its technology, but with some delay, as a commitment to consumers who incur start-up costs. We share with these papers the general insight that committing to competition limits ex-post rent extraction and may increase demand. In contrast, our focus is on mergers in vertical structures and our set-up is much less stylized, because the signing of complex vertical contracts allows the integrated monopolist to, for instance, only partially foreclose downstream competitors.

Finally, related to our work is also the literature on "divisionalization", which studies motives for which firms may sell their products through multiple downstream divisions. Baye et al. (1996) Corchón (1991), Polasky (1992) consider the case of inter-brand competition for an homogeneous product, as we do in this paper, and study a two period game. First, two upstream firms choose their *number* of downstream retailers. Second, downstream firms compete a' la Cournot. In line with Bonanno & Vickers (1988), it is shown that, while they would jointly prefer a downstream duopoly, upstream firms have an individual incentive to increase the number of downstream firms to steal profit from their competitor, which ultimately results in a competitive equilibrium downstream. As in our model, the benefit from unilaterally dividing production among competing units, relies on upstream firms credibly committing to having those downstream units behave as independent profit-maximizers.

In the next section we present the model. In section 3 we conduct our analysis, while section 4 focuses on the linear case with one single active ecosystem (Appendix B presents the results of simulations). Section 5 discusses a number of potential extensions and section 6 concludes the paper with a discussion of the recent Nvidia-ARM merger case.

[5] Consumer expectations are a crucial determinant of demand also in the vast network effects literature. For instance, in Katz & Shapiro (1985) seminal paper a firm may want to commit to additional compatible downstream competition in order to encourage more consumers to join, which boosts network effects. Hence, ecosystem effects, as we have defined them, may arise from network effects (see Padilla et al. (2021).

2 Model

A market is served by two ecosystems. Each ecosystem is composed of n downstream, consumer-facing, firms and a single upstream firm who supplies an essential input to all downstream firms in the ecosystem. The first ecosystem is controlled by upstream firm A and the second is controlled by B. We assume that the n downstream firms are identical. While each ecosystem produces a homogeneous final product, the two ecosystems sell substitute but differentiated products. While the recent economics literature on ecosystems emphasizes downstream complementarity (e.g., see **?**), our downstream product homogeneity assumptions is conservative given our aims. Foreclosure is rarely profitable if downstream producers sell differentiated products, which compete less intensely with the product of an integrated firm.

The game we study develops in three periods. In the first period (*wholesale pricing stage*), A and B choose two-part tariffs that they offer to all their downstream firms. For ecosystem $j \in \{A, B\}$ we denote $(T_i^j, c_i^j)_{i=1}^n$ the offer made, where T_i^j represents the fixed payment requested to downstream firm i and c_i^j the wholesale price, or per-unit payment. For simplicity, we assume offers are observable and downstream firms must accept or reject them, simultaneously. Decisions are also observable and, if a firm rejects, it collects its reservation payoff of zero. The marginal costs of A and B are normalized to zero.

We introduce frictions in vertical negotiations by assuming that the upstream firms are constrained in the share of profit it can extract from downstream firms in the form of fixed fees. Let $\hat{\pi}_i^j(h)$ be the continuation equilibrium payoff of firm i in ecosystem j following history h where offers have been made. We follow Calzolari et al. (2020) and assume offers will be rejected if $T_i^j > \lambda \hat{\pi}_i^j(h)$, where $\lambda \in [0, 1]$ is a parameter that measures frictions in upstream-downstream negotiations. In particular, $\lambda = 0$ corresponds to the case of pure linear wholesale prices, while $\lambda = 1$ corresponds to the traditional two-part tariffs scenario.[6]

In the second period (*between-ecosystem competition*), a (possibly unbounded) mass of final buyers decide which ecosystem to join. The decision to join an ecosystem is irreversible.[7] At this stage, consumers are ex-ante identical, but, as we discuss later, they will learn about their heterogeneous individual values before making their downstream purchases. Products are differentiated across ecosystems and, when deciding which to join, consumers

[6]See Calzolari et al. (2020) for microfoundations of this approach. For instance suppose firms are risk neutral for both pure gains and losses, but they are risk-averse when trading lotteries comprising losses and gains. Also, assume with some probability downstream firms face no demand at all and otherwise everything is as normal. Then, the problem of the upstream firm takes our form, with $1 - \lambda$ as degree of risk-aversion. If $\lambda = 1$ downstream firms are risk-neutral and all expected profits can be extracted by means of fixed fees.

[7]This assumption should not be taken literally. Switching costs need not be unbounded for our equilibrium analysis to continue to hold. Our results require the presence of substantial switching costs or the need for consumers to engage in meaningful ecosystem-specific investment.

form expectations about the downstream surplus they will obtain once product development by downstream firms is completed and competition among them takes place. Since in the downstream homogeneous-product market there's a one-to-one relationship between consumer surplus and final price, let's denote with $x^j(p_e^j, p_e^{-j}) > 0$ the volume of consumers that choose ecosystem $j \in \{A, B\}$ when they anticipate final prices to be p_e^j and p_e^{-j} in ecosystem j and $-j$. We assume that x is twice differentiable, strictly decreasing in the expected final price charged in the j ecosystem, $dx^j/dp_e^j < 0$, and strictly increasing in the expected final price charged in the other ecosystem $-j$, $dx^j/dp_e^{-j} > 0$. Furthermore, we assume that x^j is log-concave in p_e^j, enough to guarantee that the upstream firms' profit maximization problems, (O.Pre) and (O.Post) defined later in Section 3.3., have a unique solution. This condition is satisfied, for instance, in the linear case we develop in Section 4.

In the third period of the model (*within-ecosystem competition*) final prices are set in both ecosystems. Then, consumers learn about their values and purchasing decisions are made. At this stage, consumer heterogeneity generates a downward sloping demand for downstream firms. In ecosystem j, the final price p_j is determined by downstream competition between the n firms. We assume that downstream firms sell a homogeneous product and that outcomes are determined by the Cournot equilibrium condition.[8] Being identical, all firms face, in addition to the wholesale price, the same marginal cost, which we assume constant and, without loss, normalize to zero. We assume the value of each consumer is drawn from a smooth, strictly increasing and log-concave CDF F in $[0, 1]$, admitting density f. This distribution also determines beliefs that firms have about consumers' valuations, when they make their pricing decisions. Log-concavity guarantees that the profit function of downstream firms is strictly concave in the quantity produced.

We consider two variants of this game, one *pre-merger* scenario and one *post-merger*. The pre-merger scenario has been described above. In the post-merger scenario, A integrates with firm 1 and the model is as in the pre-merger case, except we maintain $c_1 = 0$ and $T_1 = 0$, which is common knowledge.[9] The assumption that upstream firms cannot commit to a wholesale price for their subsidiary reflects the fact that integration partially reduces commitment and delegation power of the firms that control the ecosystem.

In any scenario, we look at subgame perfect equilibria of this game.

[8]In contrast to Bertrand competition, Cournot equilibrium leaves homogeneous firms with a profit that can be appropriated by the upstream firm and thus represents a motive for foreclosure additional to the the attempt to enforce a specific downstream price.

[9]As long as the decision to integrate is made ex-ante, endogenizing the merger (without introducing additional frictions or benefits) simply implies evaluating whether the sum of payoffs of the upstream firm and the downstream firm that is integrated is larger pre or post-merger.

3 Analysis

We look for subgame perfect equilibria of this model by using backward induction. Hence, we start from within-ecosystem competition.

3.1 Within-ecosystem competition

Consider the last stage of the game and fix constant $x \in (0, \infty)$ the volume of consumers that has chosen one of the two ecosystems depending on previous history. (In this subsection we omit to specify superscript j since within-ecosystem competition in any one ecosystem is not affected by decisions made within the other.) At the within-ecosystem competition stage, firms sell homogeneous products and therefore a unique market price p will prevail. At that price, demand for the good produced in this ecosystem will be equal to:

$$x[1 - F(p)].$$

Let q be the total non-negative quantity sold by all firms in the ecosystem and let $q_{-i} = \sum_{k \neq i} q_k$.[10] Since F is strictly increasing, we can write the inverse demand function for $q \in [0, x]$ as

$$P(q) = F^{-1}\left(1 - \frac{q}{x}\right),$$

and, otherwise, let $P(q) = 0$ for $q > x$. Then, $(q_1, q_2, ..., q_n)$ is a *downstream Cournot equilibrium* if and only if for all $i = 1, \ldots, n$, it satisfies

$$q_i \in \arg\max_{y \in \Re^+} [P(y + q_{-i}) - c_i]y,$$

that is each firm is supplying the quantity of product that maximizes its profit, net of the already-sunk fixed fee T_i, taking as given the equilibrium supply of others firms.

It is well known that an equilibrium exists in this setting (see McManus (1964), Roberts & Sonnenschein (1976)) since log-concavity of F implies log-concavity of P and, as a consequence, strict concavity of firms' profit functions (see Bulow & Roberts (1989)). The equilibrium is also unique given log-concavity of demand and convexity of the cost functions (see von Mouche & Quartieri (2013)). Importantly, uniqueness implies that the continuation equilibrium following acceptance of offers is uniquely determined by the profile of marginal costs $(c_1, \ldots, c_n)$, with $c_i = \infty$ to indicate that the firm is not producing because it rejected an offer.

[10] Note that x determines the volume of consumers that can potentially buy (i.e., the maximum demand) while the actual quantity of consumers who purchase is determined by the equilibrium quantity decisions of downstream firms.

The next lemma shows that the equilibrium price is independent of the volume of consumers that has joined the ecosystem. While this is obvious in the monopoly case, where the monopolist maximizes $x[1 - F(p)]p$ in p taking x as given, it deserves a short proof in the case of competing firms.

Lemma 1 *Fix $c_1, \ldots, c_n$ and denote with $(q_1, \ldots, q_n)$ an equilibrium for $x = 1$. Then, $(xq_1, \ldots, xq_n)$ is an equilibrium for $x > 0$ and $P(x(q_1 + \cdots + q_n)) = F^{-1}(1 - (q_1 + \cdots + q_n))$ is the equilibrium price, which does not depend on x.*

Proof The second part of the statement is obvious once we have proved the first. A necessary condition for equilibrium for firm i is

$$\frac{q_i}{x} \frac{1}{f\left(F^{-1}(1 - \frac{q_i + q_{-i}}{x})\right)} = F^{-1}\left(1 - \frac{q_i + q_{-i}}{x}\right) - c_i.$$

If this holds for $(q_1, \ldots, q_n)$ for $x = 1$, it also holds for $(xq_1, \ldots, xq_n)$ when $x \neq 1$. $\square$

This lemma simplifies the analysis considerably. It implies that, for both ecosystems, consumers expectations of prices are not determined by the volume of consumers that join one ecosystem or another, but by market structure and the contracts that have been signed by the downstream firms.[11]

Before proceeding, we present another lemma which will be used later. While it is immediate to see that an increase in the marginal cost of a firm reduces that firm's production and increases production of other firms, we now show that under the assumed log-concavity, an increase in the cost of any individual firm also reduces *total* quantity produced.

Lemma 2 *Total quantity produced decreases when the marginal cost of any firm producing a strictly positive quantity increases.*

Proof A proof for the case of a common cost shift for symmetric firms is in Seade (1985), but it is straightforward to extend it to the case of individual shifts for heterogeneous firms (see also Dixit (1986)). Let's return to the first order conditions from Lemma 1, for $i = 1, \ldots, n$ we have

$$\frac{q_i}{xf\left(F^{-1}(1 - \frac{q_i + q_{-i}}{x})\right)} = F^{-1}\left(1 - \frac{q_i + q_{-i}}{x}\right) - c_i.$$

[11] This property differentiates further our setup by one with classic network effects. With network effects, the value that consumers expect to obtain from the ecosystem would depend on the volume, thus providing stronger incentives to lower prices.

Summing up the FOCs for all firms, assuming they all produce a non-zero quantity, the following must hold

$$nF^{-1}\left(1-\frac{q}{x}\right)-\sum_i c_i-\frac{q}{xf\left(F^{-1}(1-\frac{q}{x})\right)}=0,$$

where we write total quantity as $q=\sum_i q_i$. Using $P(q)=F^{-1}(1-q/x)$ we get

$$Z(q,c)=nP(q)-\sum_i c_i+qP'(q)=0. \tag{1}$$

We can then use the implicit function theorem to obtain

$$\frac{\partial q}{\partial c_i}=-\frac{\frac{\partial Z}{\partial c_i}}{\frac{\partial Z}{\partial q}}=\frac{1}{(n+1)P'(q)+qP''(q)},$$

which is negative because log-concavity of F implies $2P'(q)+qP''(q)<0$ and, a fortiori, $(n+1)P'(q)+qP''(q)<0$ since $P'<0$. $\square$

We conclude this section introducing the notation for a downstream equilibrium. For $x=1$, for any profiles of costs $(c_1,\dots,c_n)$ in one ecosystem, we denote the unique downstream equilibrium as $(\hat{q}_1(c_1,\dots,c_n),\dots,\hat{q}_n(c_1,\dots,c_n))$ and we maintain the mapping is continuous as it is, for instance, in the linear case. Let the equilibrium price be $\hat{p}(c_1,\dots,c_n)$ and let $\hat{\pi}_i(c_1,\dots,c_n)=\hat{q}_i(c_1,\dots,c_n)\left(\hat{p}(c_1,\dots,c_n)-c_i\right)$ indicate the equilibrium profit of downstream firm i, net of fixed fees. Quantity and profit of downstream firm i at $x\neq 1$ will simply be $x\hat{q}_i(c_1,\dots,c_n)$ and $x\hat{\pi}_i(c_1,\dots,c_n)$.

3.2 Between-ecosystem competition

We can then proceed backward to the between-ecosystem competition stage, at which consumers make their ecosystem decision choices. We henceforth reinstate the ecosystem-specific superscript and denote with x^j the share of consumers that choose ecosystem j.

Since the downstream prices for both ecosystems are, in any continuation equilibrium, equal to $\hat{p}(c_1^A,\dots,c_n^A)$ and $\hat{p}(c_1^B,\dots,c_n^B)$, consumer preferences imply that the total volume for the j managed ecosystem is equal to

$$\hat{x}^j(c_1^j,\dots,c_n^j;c_1^{-j},\dots,c_n^{-j})=x^j(\hat{p}(c_1^j,\dots,c_n^j),\hat{p}(c_1^{-j},\dots,c_n^{-j})).$$

3.3 Wholesale-pricing stage

We are now in a position to characterize the equilibrium upstream decisions, which take place at the wholesale pricing stage. Our first observation is that we can focus attention to equilibria where all offers are accepted and this requires the upstream firm j to set

$$T_i^j \leq \lambda \hat{x}^j(c_1^j, \ldots, c_n^j; c_1^{-j}, \ldots, c_n^{-j})\hat{\pi}_i(c_1^j, \ldots, c_n^j).$$

To see this, note that downstream firms always strictly prefer to accept offers such that $T_i^j < \lambda \hat{x}^j(c_1^j, \ldots, c_n^j; c_1^{-j}, \ldots, c_n^{-j})\hat{\pi}_i(c_1^j, \ldots, c_n^j)$ and are indifferent when this holds with equality. Hence, when an upstream firm j would have its offer rejected by firm i, there is another payoff-equivalent equilibrium where it sets $c_i^j = \infty$ and $T_i^j = 0$ and that offer is accepted.

In light of the above, for given actions taken in the competing ecosystem, the best-response of j in the *pre-merger* scenario solves

$$\max_{\{T_i^j, c_i^j\}_{i=1,\ldots,n}} \hat{x}^j(c_1^j, \ldots, c_n^j; c_1^{-j}, \ldots, c_n^{-j}) \sum_i c_i^j \hat{q}_i(c_1^j, \ldots, c_n^j) + \sum_i T_i^j \tag{O.Pre}$$

subject to:

$$T_i^j \leq \lambda \hat{x}^j(c_1^j, \ldots, c_n^j; c_1^{-j}, \ldots, c_n^{-j})\hat{\pi}_i(c_1^j, \ldots, c_n^j) \textit{ for } i = 1, \ldots, n$$

In the *post-merger* scenario j internalises the profit of the integrated downstream firm 1 and its best response solves

$$\max_{\{T_i^j, c_i^j\}_{i=2,\ldots,n}} \hat{x}^j(0, c_2^j, \ldots, c_n^j; c_1^{-j}, \ldots, c_n^{-j}) \left[\hat{\pi}_1(0, c_2^j, \ldots, c_n^j) + \sum_{i=2}^n c_i^j \hat{q}_i(0, c_2^j, \ldots, c_n^j)\right] + \sum_{i=2}^n T_i^j \tag{O.Post}$$

subject to:

$$T_i^j \leq \lambda \hat{x}^j(0, c_2^j, \ldots, c_n^j; c_1^{-j}, \ldots, c_n^{-j})\hat{\pi}_i(0, c_2^j, \ldots, c_n^j) \textit{ for } i = 2, \ldots, n$$

We remark that c_i^j should be interpreted as a markup over upstream marginal cost, rather than as absolute cost value. Negative values then imply that the upstream firm sells below marginal cost, not necessarily that it is making a payment to downstream firms.

The following lemma is immediate and, therefore, stated without proof. It says that it is without loss of generality to assume that all participation constraints are binding.

Lemma 3 *In a pre-merger equilibrium* $T_i^j = \lambda \hat{x}^j(c_1^j, \ldots, c_n^j; c_1^{-j}, \ldots, c_n^{-j})\hat{\pi}_i(c_1^j, \ldots, c_n^j)$ *for* $i = 1, \ldots, n$ *and in a post-merger one* $T_i^j = \lambda \hat{x}^j(0, c_2^j, \ldots, c_n^j; c_1^{-j}, \ldots, c_n^{-j})\hat{\pi}_i(0, c_2^j, \ldots, c_n^j)$ *for* $i = 2, \ldots, n$.

We are now ready to define equilibrium for the purpose of the rest of the paper. Following Lemma 3, we define the equilibrium upstream decisions pre-merger as the wholesale price offers in ecosystems A and B, $(c_1^A, \ldots, c_n^A)$ and $(c_1^B, \ldots, c_n^B)$, that simultaneously solve (O.Pre) for both ecosystems A and B. Then, define the equilibrium upstream decisions post-merger as the wholesale price offers in ecosystem A and B, $(c_2^A, \ldots, c_n^A)$ and $(c_1^B, \ldots, c_n^B)$, that simultaneously solve (O.Post) for A and (O.Pre) for B.

3.4 Main Result

In this section we present our main results. To begin with, suppose there are *no* ecosystem effects and x^j is independent of the costs charged to downstream firms, for instance $x^j = 1$ for $j = \{A, B\}$. Since there's no between-ecosystem competition and the volume of consumers is independent of expected prices, the equilibrium upstream decision in one ecosystem is determined solely by solving its own maximization problem. It is easy to see that in this case, for any $\lambda < 1$, the solution to (O.Pre) involves all firms being treated symmetrically and operating in equilibrium.[12] On the other hand, it is well known that the solution to (O.Post) involves no production and zero profit for all non-integrated firms when there are *no ecosystem effects*. There is *complete input foreclosure* post-merger. Moreover, it can be shown that post-merger the integrated firm implements the monopoly outcome. We emphasise this result in the following proposition.

Proposition 1 *Let $x^j(p_e^j, p_e^{-j}) = 1$ and $\lambda < 1$. Then (O.Post) is solved by $T_i^j = 0$, $c_i^j \geq p^M$ for $i > 1$. At a solution we have $\hat{q}_i(0, c_2^j, \ldots, c_n^j) = 0$ for $i \neq 1$ and $\hat{p}(0, c_2^j, \ldots, c_n^j) = p^M$.*

The result is conventional wisdom and we do not provide a proof. However, a couple of clarifications are in order. First, in light of Lemma 1, the result extends to the case where $x^j \neq 1$ but is independent of prices. Second, when $\lambda = 1$, even post-merger, the upstream firm is indifferent between any combination of $(0, c_2, \ldots, c_n)$ that delivers $\hat{p}(0, c_2, \ldots, c_n) = p^M$. Hence, while possible, foreclosure is not needed to maximize profits of the integrated firm when downstream profit can be fully extracted by means of two-part tariffs.

Our goal is to contrast the above with the case in which ecosystem effects are at play, that is, consumers patronize ecosystems based on their expectations of downstream prices, which are driven by the observed market structure in each ecosystem. Our main result of this section is that, under the stated assumptions, *complete input foreclosure* will not take place following the merger in the presence of ecosystem effects. At the margin, starting from the full-foreclosure outcome, the integrated upstream firm has an incentive to lower the cost of one or more downstream competitors below the monopoly level. This leads to a lower final price, thus reducing the margin for the merged firm, and, for any fixed volume, it may also lower the share of total output sold by the merged downstream firm. However, when between-ecosystem competition is taken into account, the increase in volume, which is driven away from its competitor ecosystem, benefits the merged firm.

Proposition 2 *An equilibrium exists in the post-merger scenario where the upstream firm in ecosystem j merges with downstream firm 1. In any equilibrium of the game, the integrated upstream firm chooses a symmetric wholesale price c^j such that $c^j < \hat{p}(0, c^j, \ldots, c^j) < p^M$.*

[12] If $\lambda = 1$ all firms will make zero-profit and the upstream firm essentially operates as an integrated firm, so, without ecosystem effects, it might as well shut-down some of the non-integrated downstream firms.

Proof See Appendix A. □

There's a simple mathematical intuition for this result. Rearranging the first-order condition of (O.Post), we get

$$\tilde{\pi}' = -\frac{\hat{x}^{j'}}{\hat{x}^j}\tilde{\pi}$$

where $\tilde{\pi}$ is the profit of the upstream firm when $\hat{x}^j = 1$ and $\tilde{\pi}'$ and $\hat{x}^{j'}$ are the derivatives with respect to c^j. For fixed volume, the profit of the upstream firm increases in c^j (i.e., the wholesale price to non-merged downstream firms) but it reaches a plateau when $c^j = p^M$, that is with complete foreclosure. Then, of course, raising c^j further does not increase the profit of the upstream firm. However, while the effect of lowering c^j from the full-exclusion level is negligible on the ability to extract profit for a given level of volume when ecosystem effects are muted (since $\tilde{\pi}'(p^M) = 0$), the marginal increase in profit that comes from additional volume stolen from the competing ecosystem, that is $\frac{\hat{x}^{j'}}{\hat{x}^j}\tilde{\pi}$, is non-negligible when consumers have valuations that come from a log-concave distribution. Hence, the upstream firm always benefits from setting c^j in such a way that downstream competitors produce a positive quantity. Of course, as we highlight in the next section where we consider the linear case, how low is the optimal c^j depends on how weak are the ecosystem effects and how easy is to extract downstream profits (i.e., λ).

We remark that downstream competitors are not foreclosed because the only way the upstream firm is able to commit to low-prices is by committing to contracts with downstream firms. Credible commitment to lower prices, attracts consumers to its ecosystem, who choose before downstream prices are finalized, and away from the competing ecosystem. Instead, suppose consumers could observe final prices *before* committing to one ecosystem or the other. In this case, we would have full foreclosure of downstream competitors, as the integrated upstream firm would be able to keep prices low in order to harness ecosystem effects without giving shares to competitors.

A notable special case occurs when one of the ecosystems is, at the outset, an integrated monopolist. By definition, such a monopolist is unable to make price commitment to consumers joining its ecosystems. Hence, it only represents a static competitive threat to the other open ecosystem. We discuss this case in details in the next section. For now, we conclude this section by arguing that, under a further assumption that guarantees the strategic complementaries of ecosystem decisions, the presence of active ecosystem competition represents an additional pressure toward non-foreclosure following the merger.

Proposition 3 *Assume* $\frac{d^2 x^j}{d\hat{p}^j d\hat{p}^{-j}} \geq 0$. *Suppose ecosystem j is vertically integrated with one of its downstream firms. Let $\hat{c}^j_{open}$ and $\hat{c}^j_{int}$ be the symmetric equilibrium wholesale prices charged by ecosystem j when it competes with an open ecosystem and when it competes with a fully integrated ecosystem charging p^M downstream, respectively. Then, $\hat{c}^j_{open} < \hat{c}^j_{int}$.*

Proof We prove that, given our assumption, the equilibrium strategies of ecosystem j and $-j$ are strategic complements in $c^j, c^{-j} \in (0, p^M)$. The claim follows as a corollary.

Let $\tilde{\pi}^j$ be the profit of the upstream firm in ecosystem j. Given c^{-j}, ecosystem j chooses c^j such that

$$W(c^j, c^{-j}) := \frac{d\hat{x}^j}{dc^j}\tilde{\pi}^j + \frac{d\tilde{\pi}^j}{dc^j}\hat{x}^j = 0$$

Applying the implicit function theorem, and omitting ecosystem superscripts, we obtain

$$\frac{dc^j}{dc^{-j}} = -\frac{\frac{\partial W}{\partial c^{-j}}}{\frac{\partial W}{\partial c^j}} = -\frac{\hat{x}_{c^j c^{-j}}\tilde{\pi} + \hat{x}_{c^{-j}}\tilde{\pi}_{c^j}}{\hat{x}_{c^j c^j}\tilde{\pi} + 2\hat{x}_{c^j}\tilde{\pi}_{c^j} + \hat{x}\tilde{\pi}_{c^j c^j}}$$

where the subscript denotes partial differentiation. From the second-order condition, at any equilibrium we have $\frac{\partial W}{\partial c^j} < 0$. Since we assumed $\frac{d^2 x^j}{d\hat{p}^j d\hat{p}^{-j}} \geq 0$, we have $\frac{d^2\hat{x}_j}{dc^j dc^{-j}} = \frac{d^2 x^j}{d\hat{p}^j d\hat{p}^{-j}}\frac{d\hat{p}^j}{dc^j}\frac{d\hat{p}^{-j}}{dc^{-j}} \geq 0$. Thus, the numerator is positive because $\frac{d\hat{x}_j}{dc^{-j}} > 0$ and $\frac{d\tilde{\pi}_j}{dc^j} > 0$ as $c^j < p^M$. Therefore, $\frac{dc^j}{dc^{-j}} > 0$: the two ecosystems have strategically complementary strategies. $\square$

4 The linear case

Having presented our main result, in this section we consider a simple linear setting where one of the ecosystems is an integrated monopolist, which we denote with I, and the other is an open ecosystem, U, with two downstream firms, 1 and 2. The aim is to obtain closed form solutions and engage in comparative statics.

First, in order to model within-ecosystem competition we assume consumers decide between ecosystems following a simple linear demand structure with differentiated products (e.g. see Levitan & Shubik (1972)), where the volume for the ecosystem U is given by

$$x(p_e^U, p_e^I) = 1 - (p_e^U - p_e^I)/\beta$$

with $\beta > 0$. It follows that $x' = -1/\beta < 0$. The lower β, the more the two ecosystems compete with each other, the larger are ecosystem effects.[13]

Second, we assume that the willingness to pay of individual consumers is uniformly distributed, with $F(v) = v$ for $v \in [0, 1]$. Under the uniform distribution, we have $p_e^I = p^M = 1/2$, since I is assumed to be an integrated monopolist operating with a single firm in the downstream market. Observe that the uniform distribution is log-concave and induces strictly concave profit functions and a unique equilibrium, since the associated reaction functions intersect only once.

[13] An analogous demand would also arise from an Hotelling-type model of product differentiation and price competition, e.g. see Mathewson & Winter (1984)

Equilibrium quantities and prices for $x = 1$ take the following classic form:

$$\hat{q}_1(c_1, c_2) = \frac{1 + c_2 - 2c_1}{3}; \quad \hat{q}_2(c_1, c_2) = \frac{1 + c_1 - 2c_2}{3}; \quad \hat{p}(c_1, c_2) = \frac{1 + c_1 + c_2}{3}.$$

The linear case satisfies our condition for concavity of the upstream profit functions, which establishes that an equilibrium, that is the choice of contracts of U, will indeed be symmetric. It is a matter of computation to show that the optimal symmetric c in the pre-merger case is

$$\hat{c}(\beta, \lambda) = \frac{\beta}{2} + \frac{\sqrt{\lambda^2 + 4(\lambda - 3)^2\beta^2 - 4(\lambda - 3)(\lambda - 1)\beta - 2\lambda + 13} + 3\lambda - 5}{4(\lambda - 3)}$$

while c_2 in the post-merger case is

$$\hat{c}_2(\beta, \lambda) = \frac{1}{2} + \beta - \sqrt{\beta^2 + \frac{3}{4(5 - 4\lambda)}}.$$

In line with results from the previous section, we can see that foreclosure post-merger will not be complete.

Remark 1 $\hat{c}_2(\beta, \lambda) < 1/2 = p^M$.

Also, note that $\lim\limits_{\beta \to \infty} \hat{c}_2 = 1/2 = p^M$. That is, as positive ecosystem effects disappear, i.e., there is no between-ecosystem competition and consumers are locked into the ecosystem, we are back to the classic foreclosure scenario.

These values can then be used to obtain a closed form solution to our key variables, including price and quantities. Focusing on the linear case, we can highlight the following results, which can all be easily verified with some algebra or simulations (see Appendix B).

Remark 2 *Wholesale prices increase when the intensity of competition between ecosystems (i.e., strength of ecosystem effect) is reduced (i.e., β increases), both pre and post-merger. That is $\frac{\partial \hat{c}}{\partial \beta} > 0$ and $\frac{\partial \hat{c}_2}{\partial \beta} > 0$.*

In contrast to the classic setting without ecosystem effects, when λ is sufficiently high, we may observe wholesale prices going below marginal cost of the upstream firm at lower levels of β (i.e., strong ecosystem effects), as the upstream firm attempts to harnesses ecosystem effects by inducing a low downstream price and intensifying competition with the rival ecosystem. The remark below complements the above.

Remark 3 *Wholesale prices decrease when the share of downstream profits that can be appropriated via fixed fees, λ, increases. That is $\frac{\partial \hat{c}}{\partial \lambda} < 0$ and $\frac{\partial \hat{c}_2}{\partial \lambda} < 0$.*

This is a standard result, which continues to hold with within-ecosystem competition and ecosystem effects. The easier is for the upstream firm to extract profit via the fixed fee, the more it can reduce the distortions created by positive wholesale price. The next result is more surprising.

Remark 4 *As the strength of ecosystem effects grows, the profit of the downstream competitor converges to the pre-merger scenario* $\lim_{\beta \to 0} \mid \hat{\pi}_2(\hat{c}, \hat{c}) - \hat{\pi}_2(0, \hat{c}_2) \mid = 0$

This result suggests that there might be minimal to no foreclosure following a vertical merger in an industry exhibiting strong ecosystem effects.

Remark 5 *As the strength of ecosystem effects grows, the profit of the integrated firm converges to the pre-merger scenario*

$$\lim_{\beta \to 0} \mid 2\left(\lambda \hat{\pi}_1(\hat{c}, \hat{c}) + \hat{c}\hat{q}_1(\hat{c}, \hat{c})\right) - \left(\hat{\pi}_1(0, \hat{c}_2) + \lambda \hat{\pi}_2(0, \hat{c}_2) + \hat{c}_2 \hat{q}_2(0, \hat{c}_2)\right) \mid = 0$$

This result suggests that with strong ecosystem effects there is little gain from strategic monopolization. Combining the last two remarks, we see that with strong ecosystem effects vertical integration has minor (strategic) effects on firms profitability, which is driven by the need to keep prices at a level that maximizes the ecosystem's value.

The next result focuses on market outcomes following integration, which are in general ambiguous and depend on the interplay of the exogenous parameters.

Remark 6 *For any* $\lambda \in [0, 1]$ *there exists non-decreasing* $\frac{4\sqrt{2}-3}{2} \geq \tau_p(\lambda) \geq 0$ *such that for all* $\beta > \tau_p(\lambda)$*, the post-merger final price is* below *the pre-merger price (equivalently* $\hat{c}_2 \leq 2\hat{c}$*), and consumers benefit from the merger as a result. Otherwise, for* $\beta < \tau_p(\lambda)$*, the price increases post-merger and consumers are worse off.*[14]

Note that $\tau_p(\lambda) = 0$ for $\lambda \leq 1/2$. Hence, when downstream surplus extraction by means of two-part tariffs is sufficiently imperfect, the merger always benefits consumers. It turns out that the merging firms are also better off, while the downstream competitor is worse off. This observation and a description of welfare outcome for the case $\lambda \geq 1/2$ is contained in the next remark.

Remark 7 *a) For* $\lambda \leq 1/2$*, the merger makes consumers and the merging firms better off, while the downstream competitor is made worse off.*

b) For $\lambda > 1/2$*, there exists thresholds* $\tau_p(\lambda) > \tau_U(\lambda) > \tau_2(\lambda)$ *such that*

(b1) for $\beta > \tau_p(\lambda)$ *post-merger price is below pre-merger while for* $\beta > \tau_p(\lambda)$ *it is above;*

[14] $\tau_p(\lambda) = 0$ for $\lambda \leq 1/2$ and for $\lambda \geq 1/2$ we have $\tau_p(\lambda) = \frac{2\sqrt{\lambda^2+2\lambda-1}}{5-4\lambda} - \frac{2\lambda+1}{2(5-4\lambda)}$

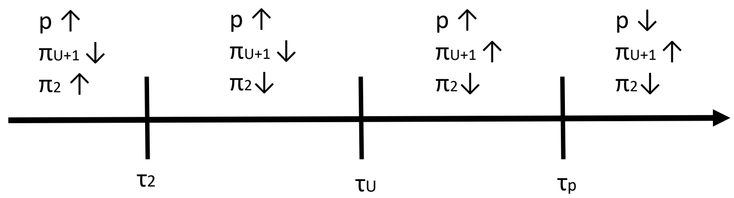
(b2) for $\beta > \tau_U(\lambda)$ the merger is profitable and for $\beta < \tau_U(\lambda)$ the merger is unprofitable;

(b3) for $\beta > \tau_2(\lambda)$ firm 2 is worse off and for $\beta < \tau_2(\lambda)$ is better off than pre-merger;

So, for $\lambda < 1/2$ or $\lambda > 1/2$ and $\beta > \tau_p(\lambda)$ the merger benefits consumers and merging firms, while the downstream competitor is worse off. For $\beta < \tau_2(\lambda)$, the merger makes consumers and the merging firm worse off, while making the downstream competitor better off. [15]

To simplify the interpretation of this proposition, the case of $\lambda > 1/2$ is summarized in the figure below. Note that $\tau_p(\lambda)$ is below 1.33, so it is somewhat small in relative terms.

Figure 1: Effects of the merger on price, profit of the merged firms, π_{U+1}, and downstream competitor, as a function of β for $\lambda > 1/2$.



The intuition for this result is as follows. When $\lambda < 1/2$, the inability to extract revenues through fixed fees implies that the upstream firm needs to keep wholesale prices high enough to earn profits. Then, pre-merger, wholesale prices are always above-cost. In this case, integration, which pushes to zero the internal wholesale price for the integrated firm, can only increase the ability of the upstream firm to steer volume to the ecosystem post-merger. Because post-merger the upstream firm internalizes more of the total profit from driving consumers to the ecosystem, integration provides incentives to boost ecosystem effects and to lower price further. As a result of this, and of the fact that double marginalization within the integrated firm is eliminated, consumers benefit. The upstream firm also benefits because it can always replicate the pre-merger price while making a higher profit, by raising the wholesale price and steering quantity away from the competitor. This explain why the competitor is worse off from the merger.

Instead, consider the case where the upstream firm can extract most of the downstream profit through a fixed fee (i.e. $\lambda > 1/2$). In this case, we need to distinguish between strong and weak ecosystem effects, or in other words, high or low level of between-ecosystem competition intensity. With weak ecosystem effects, wholesale prices are above cost pre-merger and the effects are the same as those outlined in the previous paragraph. Instead, when ecosystem effects are strong, the upstream firm has, pre-merger, an incentive to bring wholesale prices below cost, in order to stimulate ecosystem effects. Now, when integration takes

[15] Remarkably, for $\lambda > 1/2$ and $\beta \in (\tau_2, \tau_U)$ all economic agents are worse off from the merger. The merger is profitable while consumers are worse off only when $\beta \in (\tau_U, \tau_p)$.

place, the integrated firm loses its ability to commit to giving below-cost prices to the merged downstream firm. Hence, ceteris paribus, integration results in an increase in price. Then, in order to harness ecosystem effects again and steal volume from the competing ecosystem, the upstream firm has post-merger an incentive to lower wholesale price for the downstream competitor further than pre-merger levels. However, since because of classic business stealing the upstream firm now gains from reducing quantity of downstream competitor firm, the price may not return down to the pre-merger level (i.e., when $\beta < \tau_p$). Hence, consumers may end up being worse off. When this happens, the integrated firm tends to be also worse off from the merger (i.e., when $\beta < \tau_U$), due to the binding loss of commitment power. For the reasons outlined above, instead, the downstream competitor tends to benefit from the merger (i.e., when $\beta < \tau_2$).

The table below summarises the effects of integration to the various parties (integrated $U + 1$ and 2), depending on the strength of both intensity of within-ecosystem competition (i.e., ecosystem effects) and upstream/downstream frictions.

Table 1: Effects of Vertical Integration: pre-merger vs post-merger payoff

	Weak Effects ($\beta > \tau_p$)	*Strong Effects* ($\beta < \tau_2$)
Low frictions ($\lambda > 1/2$)	Consumers $\uparrow$, $U + D_1 \uparrow$, $D_2 \uparrow$	Consumers $\downarrow$, $U + D_1 \downarrow$, $D_2 \uparrow$
High frictions ($\lambda < 1/2$)	Consumers $\uparrow$, $U + D_1 \uparrow$, $D_2 \downarrow$	Consumers $\uparrow$, $U + D_1 \uparrow$, $D_2 \downarrow$

For completeness, we present graphically in Appendix B the results of simulating the main equilibrium variables where we vary the strength of ecosystem effects for three scenarios: (1) $\lambda = 0$ pure linear pricing, (2) $\lambda = 1/2$ imperfect two part tariffs and (3) $\lambda = 1$ unrestricted two-part tariffs. To serve as reference, we remind that in the linear case the monopoly consumer surplus 1/8 and the monopoly price is 1/2.

5 Extensions

Bertrand within-ecosystem competition. Suppose downstream firms compete in prices. Before integration, the upstream firm can control the downstream price by setting a common wholesale price equal to the price it desires. Moreover, it will extract all the downstream profit. While it would be able to implement the monopoly outcome, by the same argument we made for Cournot competition, the upstream firm will want to lower wholesale prices below the monopoly level, to steal demand away from the integrated ecosystem. Let's now suppose the upstream firm merges with one of the downstream firms. Nothing changes from the perspective of the integrated firm, it can still enforce the desired price by charging wholesale prices to the downstream firms. In short, as is known, when the downstream market is fully competitive the upstream monopolist can implement its desired outcome and extract all profit, both before and after the merger. As such, the merger has no effect on consumers or the downstream firms alike. This case is equivalent to the one where unrestricted two-part tariffs can be demanded by the upstream firm by means of take-it-or-leave-it offers.

Differentiated within-ecosystem products. We have assumed that the downstream firms sell a homogeneous product. While often unrealistic for typical ecosystems, this assumption provides the best-shot at foreclosure taking place. The analysis in this paper would go through, a fortiori, if downstream products were differentiated. To accommodate this case, the model could be extended by assuming consumers have a randomly distributed taste for variety and choose one ecosystem or the other based on the expected average price they face downstream.[16] The existence of multiple products produced by different firms would unambiguously reduce incentives to foreclose post-merger. In fact, compared to the homogeneous product case, foreclosure now shrinks the surplus that can be extracted.

Lack of contract observability. We motivated the existence of ecosystem effects by assuming that consumers opt into an ecosystem after having observed some signal about the contracts signed by downstream firms. Moreover, once they join an ecosystem, they become captive, at least in the medium term. While it is often the case that upstream firms engage in actions to publicize their vertical contracts (e.g., Apple and Google often do), the assumption of observability of contracts cannot simply be completely disposed of without affecting equilibrium. If consumers cannot observe contracts at all, then they must join an ecosystem or the other independently of the upstream/downstream contractual choices. In this scenario, however, the upstream firm will fully foreclose downstream competitors. Anticipating this, consumers will expect both ecosystems to price monopolistically. This implies that both ecosystems will do so, and integration will be followed by foreclosure. Hence, we conclude, complete lack of observability prevents the upstream firm from harnessing ecosystem effects. This is equivalent to the effect of lack of observability of the investment in classic hold-up situations (e.g., see Gul (2001) and Condorelli & Szentes (2020)).

6 Industry Application

In many ecosystems, consumers incur sizable switching costs when moving from one ecosystem to another, and often make irreversible ecosystem-specific investments ahead of the realization of final prices. This creates a hold-up problem once membership in one ecosystem has been taken up. When the firm running an ecosystem is unable to commit to final

[16] For instance, assume the individual consumer has valuations for two products distributed according to joint CDF $F(v_1, v_2)$. Then, within ecosystem demand function for product $i = 1, 2$ would be computed as $D_i(p_1, p_2) = \Pr\{v_i - p_i > 0, v_i - p_i > v_{-i} - p_{-i}\}$.

prices early on, which may happen for a variety of reasons, it will use upstream contracts to signal consumers will not be held up in their ecosystem. Hence, an upstream firm that controls an ecosystem may find it useful, as a way of competing more effectively with other ecosystems, to commit to serving several downstream firms operating in competition with each other. This will attract consumers by providing them with some confidence that the final price will be kept low by competition.

This story, which we formalised in the paper, applies, for example, to the semiconductor industry. Chip-makers produce chips based on a specific "architecture" and sell them to manufacturers of final goods, such as laptop computers and smart-phones, that need a Central Processing Unit (or CPU). Each popular architecture – e.g., Intel's x86 or Arm's one – defines a separate ecosystem. Before purchasing the chips embedded in their devices, manufacturers have to invest in developing software and capabilities that are only compatible with a specific architecture. This may occur before their purchases from chip-makers are finalized, for example because final products and the chips used in them are typically developed in parallel. Because switching architectures requires sustaining start-up costs anew, the decision to join one or another architecture-based ecosystem commits manufacturers for the medium/long term and exposes them to the risk of being held-up ex-post. Not knowing final chip-set prices, we expect manufacturers to rely on signals, such as the number of chip-makers that have received a license to employ the specific processor architecture, among other considerations, to assess which ecosystem to join. Unsurprisingly, Arm's CPU architecture has always being licensed to third-party chip-makers and, while Intel's x86 architecture was closed until recently, it was nonetheless licensed to AMD, thus ensuring some degree of within-ecosystem competition.

One conclusion that can be drawn from our analysis is that input-foreclosure concerns may be overstated for vertical mergers in industries characterized by ecosystem effects. Such concerns led Nvidia to abandon its proposed acquisition of Arm in 2022. Indeed, in December 2021, the United States Federal Trade Commission (FTC) filed a lawsuit to prevent Nvidia from acquiring Arm as the purchase would allegedly give Nvidia control over Arm's architecture that Nvidia's rivals rely on to develop competing chips. Similar concerns were raised by the United Kingdom Competition Markets Authority (CMA) and the European Commission. The findings in this paper question the assessment of the risk of foreclosure in the Nvidia/Arm merger. Given the ecosystem effects characterizing the markets where Arm and Nvidia operate, Nvidia should have realized any pricing or non-pricing initiative aimed at foreclosing access to Arm's infrastructure would have been self-defeating, especially given Intel's contemporaneous announcement to open up the x86 architecture to manufacturers other than AMD.

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Appendix A: Proof of Proposition 2

The proof of Proposition 2 is a corollary of the two following Lemmas.

Lemma 4 *There exists some $\varepsilon > 0$ such that for any c_i with $\hat{p}(0, c_2, \ldots, c_n) > c_i$ (i.e., as long as non-integrated downstream firm i has a strict incentive to produce), we have $\frac{d\hat{x}}{dc_i}(0, c_2, \ldots, c_n) \leq -\varepsilon < 0$. For any c_i such that $\hat{p}(0, c_2, \ldots, c_n) < c_i$ (i.e., as long as non-integrated downstream firm i has a strict incentive to shut down production), we must have $\frac{d\hat{x}}{dc_i}(0, c_2, \ldots, c_n) = 0$.*

Proof of Lemma 4 Observe $\frac{d\hat{x}}{dc_i} = x' \frac{d\hat{p}}{dc_i}$ where $x' < 0$ is the derivative of x with respect to P^j. Note that $\frac{d\hat{p}}{dc_i} = P' \frac{dq}{dc_i} > 0$ since $P' < 0$ and $\frac{dq}{dc_i} < 0$ by Lemma 2, as long as non-integrated firm i produces, that is when $0 < \hat{p} < p^M$ and $c_i < \hat{p}$. Furthermore, we claim that $\frac{d\hat{p}}{dc_i} \geq \varepsilon_1$ for some $\varepsilon_1 > 0$.

First, $\frac{d\hat{p}}{dc_i} = \frac{1}{(n+1)+qP''(q)/P'(q)}$ is well-defined and $\frac{d\hat{p}}{dc_i} > 0$ as $\frac{qP''(q)}{P'(q)}$ is bounded below by -2 because F is log-concave. Then, note that $\frac{qP''(q)}{P'(q)} = \frac{q}{x} \frac{f'(P(q))}{f(P(q))^2}$ is bounded above as $f'(P)$ is bounded since F is smooth and we can find $\varepsilon_2 > 0$ such that $f(P) > \varepsilon_2$ for all $P \in (0, p^M)$ as F is strictly increasing and smooth so f cannot tend down to 0 at any $P^j \in [0, p^M]$. Thus, there exists $\varepsilon_1 > 0$ such that $\frac{d\hat{p}}{dc_i} \geq \varepsilon_1$. Similarly, x' is bounded above by some $-\varepsilon_3 < 0$ for any $P^j \in (0, p^M)$ as $x' < 0$ is continuous. Therefore, $\frac{d\hat{x}}{dc_i} \leq -\varepsilon_1 \varepsilon_3$ when $0 < \hat{p} < p^M$ and $c_i < \hat{p}$, and the bound is identical for all i that has a strict incentive to produce.

If a non-integrated downstream firm i is not producing, then marginally changing c_i has no effect on the quantity produced nor on the expected price. □

Lemma 5 *At the optimum of (O.Post) the upstream firm chooses a symmetric wholesale price c^j such that $c^j < \hat{p}(0, c^j, \ldots, c^j) < p^M$ for any given $(c_1^{-j}, \ldots, c_n^{-j})$.*

Proof of Lemma 5 We develop the argument for two firms, but it readily extends to the n firms case. Consider (O.Post). Substituting for the binding participation constraint and collecting $\hat{x}_j$ the objective function becomes

$$\hat{x}_j(0, c_2^j; c_1^{-j}, c_2^{-j})[\tilde{\pi}_1(0, c_2^j) + c_2^j \hat{q}_2(0, c_2^j) + \lambda \tilde{\pi}_2(0, c_2^j)].$$

Differentiate the objective function with respect to c_2^j to get

$$\frac{d\hat{x}_j}{dc_2^j} \tilde{\pi}_j + \frac{d\tilde{\pi}_j}{dc_2^j} \hat{x}_j \tag{2}$$

where $\tilde{\pi}_j$ is the profit of the integrated upstream firm when $\hat{x}_j = 1$. First, consider $\frac{d\tilde{\pi}_j}{dc_2^j}$. Economizing on notation, the first order derivative is

$$\frac{d\tilde{\pi}_j}{dc_2^j} = \hat{\pi}_1' + c_2^j \hat{q}_2' + \hat{q}_2 + \lambda \hat{\pi}_2'$$

where the derivatives are over c_2^j. Recall that $\hat{\pi}_i(c_1^j, c_2^j) = \max_y [P(y + \hat{q}_{-i}(c_1^j, c_2^j)) - c_i^j]y$. Differentiating the downstream profits with respect to c_2^j, the *envelope theorem* implies $\hat{\pi}_1' = P'\,\hat{q}_2'\,\hat{q}_1$ and $\hat{\pi}_2' = P'\,\hat{q}_1'\,\hat{q}_2 - \hat{q}_2$. Cournot optimality conditions for both firms require $P'\hat{q}_1 = -P(q) = P'\hat{q}_2 - c_2^j$. Recalling that $\hat{p} = P(\hat{q})$, the first-derivative is now

$$\begin{aligned}\frac{d\tilde{\pi}_j}{dc_2^j} &= P'\,\hat{q}_2'\,\hat{q}_1 + c_2^j \hat{q}_2' + \hat{q}_2 + \lambda P'\,\hat{q}_1'\,\hat{q}_2 - \lambda \hat{q}_2 \\ &= -(\hat{p}(0, c_2^j) - c_2^j)(\hat{q}_2'(0, c_2^j) + \lambda \hat{q}_1'(0, c_2^j)) + (1-\lambda)\hat{q}_2(0, c_2^j).\end{aligned}$$

Observe that $\hat{p}(0,0) > 0$ and that $\hat{p}(0, c_2^j)$ is increasing in c_2^j when $\hat{p}(0, c_2^j) > c_2^j$ as we argued in the proof of Lemma 4. Also, $\hat{q}_2' + \lambda \hat{q}_1' < 0$ by Lemma 2. Hence $\frac{d\tilde{\pi}_j}{dc_2^j} > 0$ when $\hat{p}(0, c_2^j) > c_2^j$ and $\frac{d\tilde{\pi}_j}{dc_2^j} = 0$ when $c_2^j \geq \hat{p}(0, c_2^j) = p^M$.

Recall from Lemma 4 that $\frac{d\hat{x}_j}{dc_2^j}(0, c_2^j) = 0$ in the open region of costs where unintegrated downstream firms shut down production. Also, observe that $\frac{d\tilde{\pi}_j}{dc_2^j} = 0$ in the same region. Therefore, Equation 2, the first order derivative of the objective function evaluated at $c_2^j > \hat{p}(0, c_2^j) = p^M$ is equal to 0.

Let's look at the other case where the unintegrated downstream firms produce a strictly positive quantity. Let $c_2^j < p^M$ increases to $p^M - 0$, we get $\frac{d\tilde{\pi}_j}{dc_2^j} = 0$ as the previous case but $\frac{d\hat{x}_j}{dc_2^j} < 0$ with strict inequality by Lemma 4. Finally, $\hat{x}_j > 0$ and $\tilde{\pi}_j > 0$. Hence, Equation 2 evaluated as c_2^j tends up to p^M is strictly negative. Therefore, we conclude that an optimum $c_2^j < \hat{p} < p^M$. Complete foreclosure is not a best response in any case.

Next, we argue that the optimum must be symmetric. For any asymmetric wholesale price offer $(c_2^j, \ldots, c_n^j)$, it is dominated by a symmetric offer $c^j = \frac{\sum_{i=2}^n c_i^j}{n-1}$. Following Equation 1, the total Cournot equilibrium quantity $\hat{q}$ is invariant to $(c_2^j, \ldots, c_n^j)$ as long as $\sum_{i=2}^n c_i^j$ is constant and so are the price $P(\hat{q})$ and the demand for the ecosystem $\hat{x}$. Consider (O.Post) again. Factoring out $\hat{x}$, the profit function becomes

$$(1-\lambda)\hat{\pi}_1 + (1-\lambda)\sum_{i=2}^{n} c_i^j \hat{q}_i + \lambda P(\hat{q})\hat{q}$$

The first and the third terms are constant as $\hat{q}$ and $\hat{q}_1$ do not change. Rewriting the first order condition of the downstream firm in Equation **??**, we have

$$\hat{q}_i = xf(P(\hat{q}))(P(\hat{q}) - c_i^j)$$

So the objective is at maximum when $\sum_i (c_i^j)^2$ is at minimum, which is when c_i^j is the same for all i. Hence, the optimum must be symmetric. $\square$

Equipped with the two Lemmas can now complete the proof of Proposition 2. We assumed that x_j is log-concave in P^j enough so that the objective functions in (O.Pre) and (O.Post) are log-concave in c^j thus quasi-concave. So (O.Pre) and (O.Post) admit unique solutions, and the maximizers characterize the best response of one ecosystem against its competitor.

Following Lemma 5, the best response is a symmetric contract and is either at the corner $c^j = 0$ or the interior $(0, p^M)$ satisfying the first order condition. Hence it is sufficient to consider symmetric equilibrium only. The best responses are upper hemicontinuous by the maximum theorem and thus continuous because the solutions to (O.Pre) and (O.Post) are unique. Let's denote the best response of one ecosystem by $f : \mathbb{R}^+ \to [0, p^M)$ and that of the other by $g : \mathbb{R}^+ \to [0, p^M)$. For the sake of contradiction, assume that the image of $f([0, p^M))$ and the pre-image of $g([0, p^M))$ do not cross in $[0, p^M)^2$. Then, it must be either that $g(f(0)) > 0$ and $g(f(p^M)) \geq p^M$ or that $g(f(0)) < 0$ and $g(f(p^M)) \leq p^M$, which contradicts Lemma 5.

Hence, the log-concavity of x_j guarantees the existence of an equilibrium, and the equilibrium contracts c^j, c^{-j} fall within $[0, p^M)$.

Appendix B: Simulations

Figure 2: $\lambda = 0$ — Outcomes as function of β, Pre-merger in blue and Post-merger in orange.

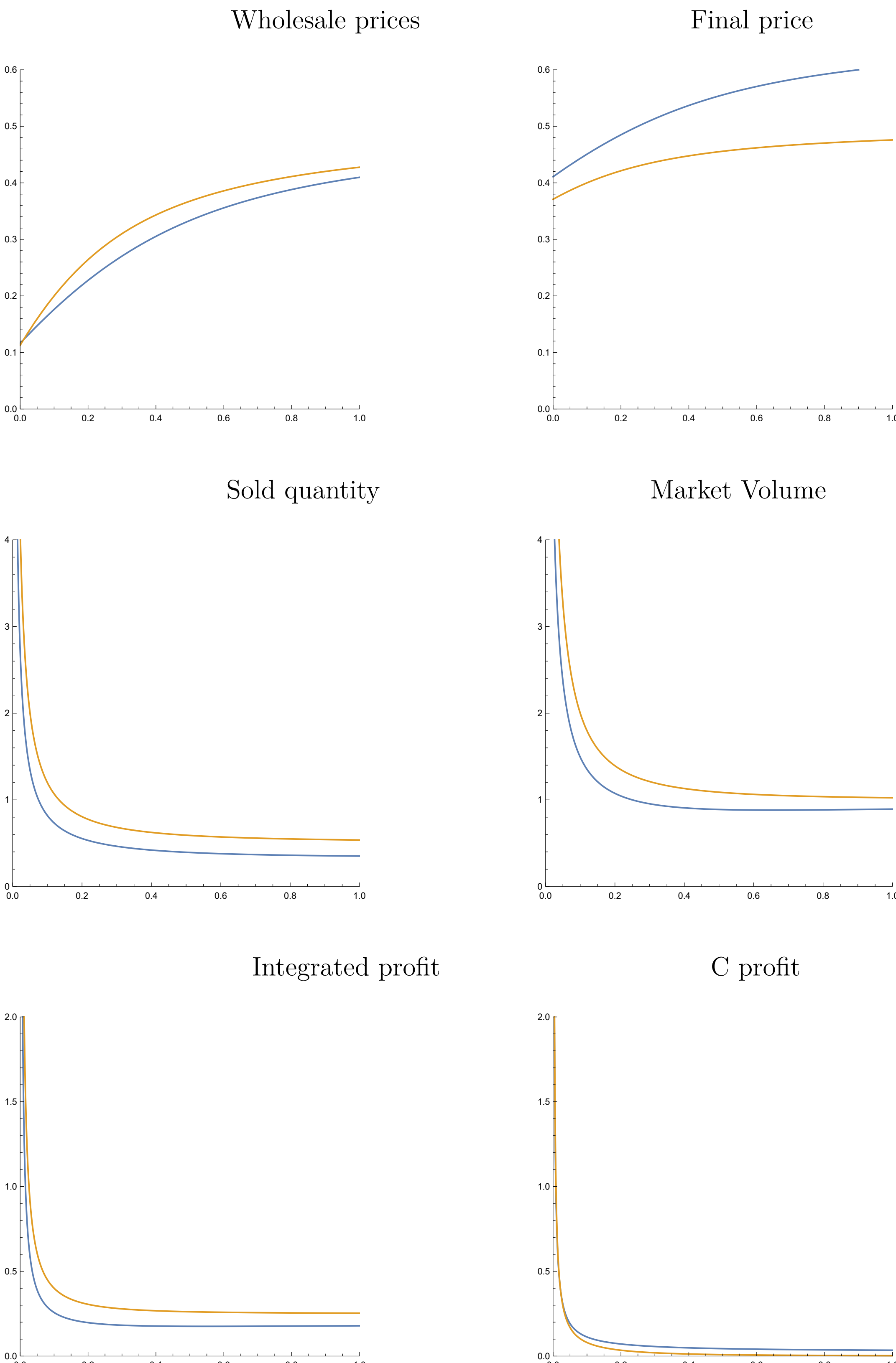

Figure 3: $\lambda = 1/2$ — Outcomes as function t, Pre merger in blue and Post-merger in orange.

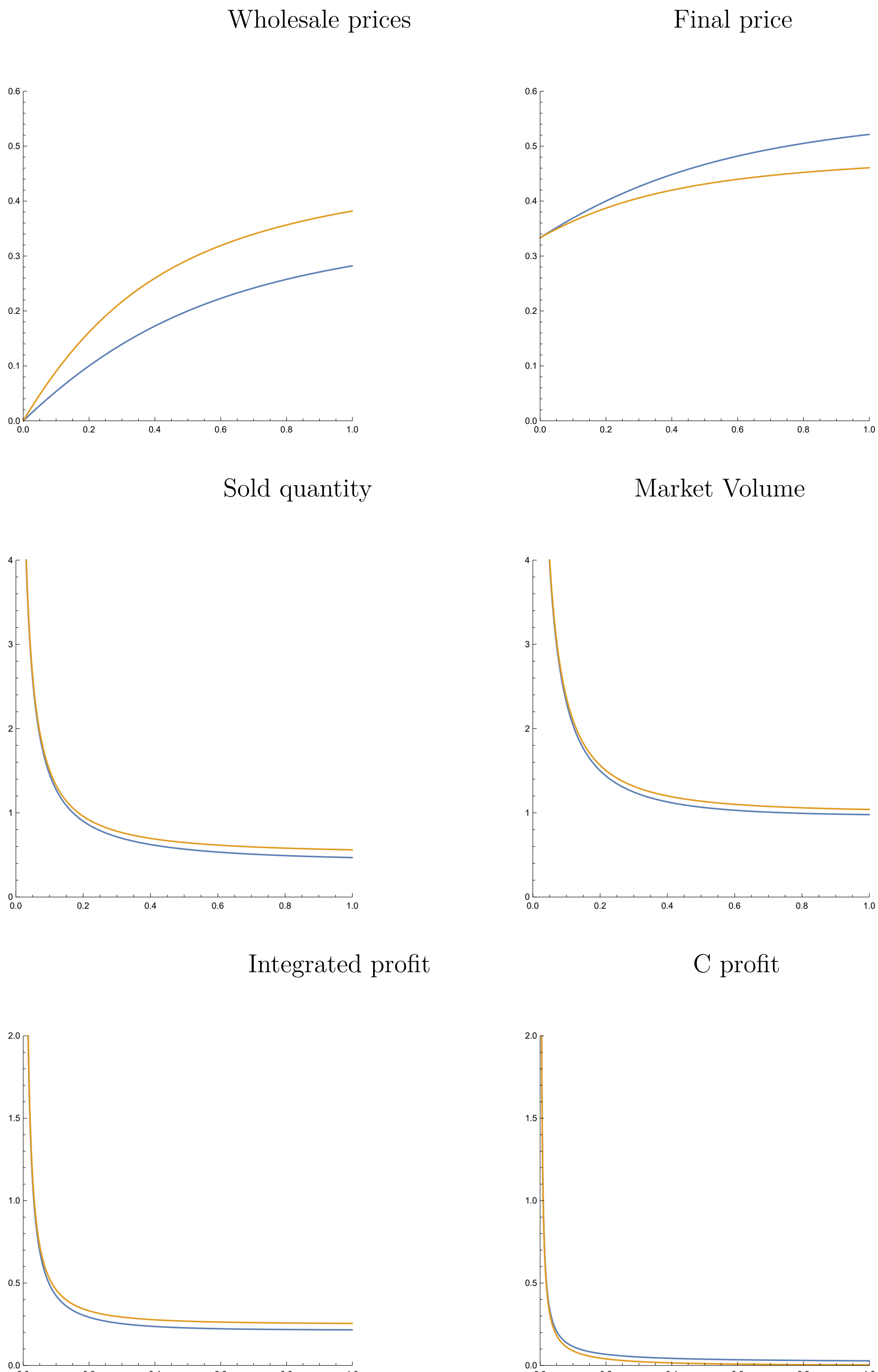

Figure 4: $\lambda = 3/4$ — Outcomes as function of β, Pre-merger in blue and Post-merger in orange.

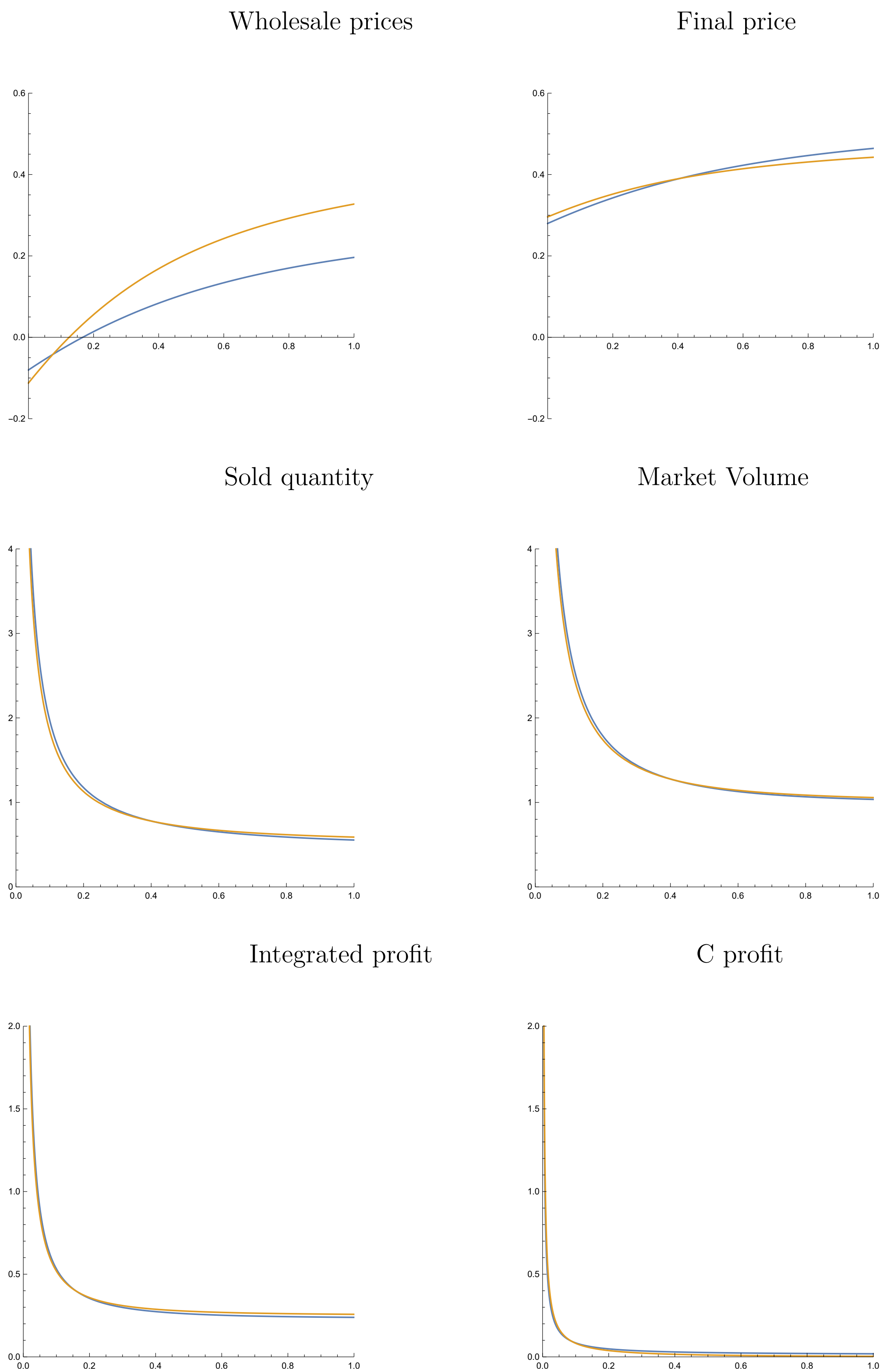

Figure 5: $\lambda = 1$ — Outcomes as function of β, Pre-merger in blue and Post-merger in orange.