A Model of Dealer Networks

Incomplete information trading over a random network

Daniele Condorelli[*], Andrea Galeotti[†], Ludovic Renou[‡]

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Abstract

We develop a model of dealer networks. A single object is traded in a network of dealers, who have private information about their private value for it. The number of connections that each dealer has is random and determined, once they purchase the object, by a common degree distribution. Sellers have full bargaining power and post optimal mechanisms to potential buyers in their neighborhood. The game proceeds until a dealer takes the object out of the market. We compute payoffs of both the initial seller and dealers. Then, we perform comparative statics, changing the network's degree distribution. We show that, typically, dealers are worse off and initial sellers better off by changes that make the network more centralized and homogeneous.

[*]Department of Economics, University of Essex, Colchester, UK.

[†]London Business School, London, UK.

[‡]School of Economics and Finance, Queen Mary University of London, London, UK.

1 Introduction

Producers do not always have the ability to reach all potential buyers and, often, better connected agents specialize in purchasing in order to resell. We speak of "dealership market", when, in addition to the above, contractually structured wholesaler-retailer relationships are absent. Our analysis focuses on the subset of dealership markets where inter-dealer resale has a prominent role, which we call, for short, "dealer networks". A key feature of dealer networks is that a dealers' willingness to pay is not driven, exclusively, by their individual valuation for taking the object out of the market. Instead, it also depends on the willingness to pay of other dealers to whom they can resell, and so on. In short, in dealer networks, willingness to pay are interdependent and influenced by the network of resale opportunities.

Dealer networks are common in over-the-counter (OTC) financial markets, such as the market for bonds and complex derivatives.[1] In OTC markets, transactions are bilateral, dealers often take securities in their inventory, and inter-dealer resale is frequent. For instance, exploiting a dataset of US municipal bonds transactions with dealer identifiers, Li and Schürhoff (2019) provide evidence that more than 20% of trades involve two or more dealers and that their relationships are long-lived.

In this paper we introduce a model of dealer networks that provides an answer to the following question. How the network architecture interacts with the bargaining protocol to affect efficiency and the distribution of surplus between different groups of agents (e.g., investors and dealers)?

We characterize equilibrium in dealer network games where (i) dealers have little information about the network structure, which is summarized by a degree distribution, (ii) ownership of the scarce good confers bargaining power and (iii) outcomes are shaped by asymmetric information about resale opportunities. Then, we perform comparative statics on how efficiency and payoffs of different agents are affected by global changes in the structure of the dealer network.

We find that a shift in the network that makes it more connected typically brings about an increase in welfare and producer surplus, but a decrease in the profits of dealers. Hence, an increase in market centralization benefits producers and harms dealers. In contrast, as the network becomes more heterogeneous in terms of connectivity, while keeping average connectivity constant, welfare and producer surplus decrease, whereas dealers' profits increase.

Our results formalize a common argument put forward by practitioners and policy makers to

[1] Dealers networks also play a key role in the market for artworks, agricultural goods, used-goods, tickets, etc.

explain why decentralized OTC markets remain pervasive, despite the well-known efficiency of centralized markets and regulatory pressure that ensued after the 2008 financial crisis.[2] That is, more decentralized and heterogeneous markets allocate to dealers a higher share of the surplus than what they would obtain in a more centralized and uniform ones.

Outline of the model and results — Our trading game starts with the producer of a single good (e.g., the issuer of a security) facing a random number of potential dealers. The realized number of dealers is drawn from an arbitrary discrete degree distribution $\pi = (\pi_1, \pi_2 \dots)$, which entirely characterizes the network structure. Each dealer is of one of two types, and the type is their private information. In particular, with probability μ, a dealer has a captive high-value client to whom it can resell the good at price v_H. With the remaining probability, the dealer has a low-value client, to whom it can sell at price v_L.

Abstracting for a moment from the specific protocol, there are two possible outcomes of the first-period sale. If it does not take place, then we assume the producer exercises an outside option, whose value is also normalized to be v_L. If it takes place, then the producer collects the payment from the buying dealer and transfers the good. Then, the new owner either resells it to its captive client or resells it downstream, to a number of dealers which is, again, randomly drawn from π. And so on, until the good is sold to a captive client of some dealer.

In our model, a network is summarized by its degree distribution, common to all traders. Hence, the match of sellers with potential buyers is an hybrid between the case of an entirely static network (e.g., as in Condorelli et al. (2017)) and the case of anonymous random matching (e.g., as in Duffie et al. (2005)). For OTC markets, this modeling choice is not inconsistent with the available empirical evidence, which points to trading relationships that are not random but not completely stable either (e.g., see Li and Schürhoff (2019) or Di Maggio et al. (2017)).

Our dealer network is a seller's market. That is, we envisage a situation in which agents who are selling the good, both producers/issuers when selling to dealers and dealers when reselling to other dealers, have full bargaining power. Hence, as standard in modeling this boundary case in an environment with asymmetric information, we assume that all sellers are able to commit to a

[2] For example, the Financial Stability Board states that "The central clearing of standardised OTC derivatives is a pillar of the G20 Leaders' commitments to reform OTC derivatives markets in response to the financial crisis" (see https://www.fsb.org/wp-content/uploads/P070818.pdf).

selling mechanisms of their choosing. Except, as usual, the chosen mechanism cannot force buyers to participate.

A key feature of this model is that a dealer's continuation value from purchasing the object is endogenous. A dealer with a high-value client will resell to it. It's continuation value is equal to v_H. A dealer with a low-value client will always attempt to resell downstream. Let's denote with V its continuation value. Hence, a stationary equilibrium (i.e., where all sellers use the same strategy) is characterized by (1) an optimal selling mechanism, used by all sellers with value v_L, to sell to a random number of buyers all having valuation in $\{V, v_H\}$ (with v_H having probability μ and V having probability $1 - \mu$); (2) best-responding bidding behavior of dealers, of high and low-value; and (3) that V is a seller's continuation value arising from (1) and (2) above.

We show that in the unique stationary equilibrium there are only two mechanisms sellers can possibly use, depending on parameters. One, sellers may use second-price auctions with reserve price equal to V. We call this format "inclusive auction", because no type of dealer is excluded and the sale takes place with probability one. In this case, high-value dealers expects a positive profit, their information rent. Two, sellers may post a fixed price equal to v_H and only sell to high-value dealers. We call this mechanism "exclusive posted price".

The characterization of unique equilibrium payoffs allows us to produce comparative statics, following Galeotti et al. (2010), on the effect of changes in the network structure (i.e., the degree distribution) on payoffs of both the initial producer and a representative dealer. First, we consider the effects of a first-order stochastic shift in the degree distribution. We show that this change always raises total welfare and the payoff of the initial seller. However, it decreases the payoff of dealers, unless the shift changes the equilibrium mechanism from exclusive posted price to inclusive auctions. Second, we consider the effects of a mean preserving spread in the degree distribution. We show that such a shift in distribution reduces the payoff of the initial seller and total welfare but raises that of dealers. The latter happens unless, again, there is a switch in the mechanism used by sellers and they start using exclusive posted prices while they were before using inclusive auction.

Literature Review — Our paper contributes to a growing literature on trading and bargaining in networks, surveyed in Condorelli and Galeotti (2016) and Manea (2016). A non exhaustive list includes Blume et al. (2009), Choi et al. (2017), Condorelli et al. (2017), Gale and Kariv (2007), Kotowski and Leister (2019), Manea (2018), Nava (2015), Siedlarek (2015). The general aim of this literature is understanding how the network determines the terms of trade, including welfare, payoffs and prices. In all these papers, a network is described as a graph (i.e., a list of nodes and edges between them). Our key methodological contribution consists in reducing the dimensionality of the exogenous variables, by representing a network through its degree distribution. This allows to perform comparative statics about changes to the global network structure. In contrast, the existing literature focuses on local changes, for instance the addition of a single link.

One strand of the literature studies models of OTC markets where asymmetric information and the network structure play a central role, as in our work.[3] An incomplete list of relevant papers includes Babus and Kondor (2018), Colliard and Demange (2018), Glode and Opp (2016), Glode and Opp (2018),Gofman (2011), Malamud and Rostek (2017). A common theme of these papers is that decentralized markets can be more efficient than centralized ones, due to specific frictions that are alleviated by decentralization. Hence, this literature provides a justification for decentralized markets based on efficiency considerations. We propose a complementary rationale for the persistence of decentralized markets. In our model a change of the market structure towards a more centralized market increases welfare, but might be nonetheless blocked by a specific group of agents, the dealers, who lose out from centralization.

2 Model

We study a game of trading over a single object that develops across a, possibly infinite, number of rounds and ends when consumption takes place. We describe the sequence of events and actions within an arbitrary period of trade.

Seller and buyers. Starting with $t = 0$, each period t is played by a seller, always denoted 0 within the period, and a new set of n^t traders, $1, 2, ..., n^t$, which enter the game in t and are buyers in the period. The number n^t is independently drawn in each t from the set of positive natural numbers, according to the probability distribution $\pi = (\pi_1, ...\pi_n, ...)$, where π_n indicates the probability that there are n new traders. Let Π be the set of all such distributions with full

[3] Another set of papers has built models of OTC markets based on random matching, following Duffie et al. (2005). For instance, in Hugonnier et al. (2019) dealers are differentiated by inventory costs and stochastic intermediation chains arise endogenously as a result.

support and finite expectation (i.e., $\pi_n > 0$ for $n \geq 1$ and $\sum_n n\pi_n < \infty$). Each new trader i has a value v_i for taking the object out of the market, which is either high or low. The value is each trader's private information and, ex-ante, it is equal to v_H with probability μ, with $1 > \mu > 0$. We assume that the *initial seller*, the seller who owns the object at $t = 0$, has value v_L for it.

Trading. After observing the realization n^t, the owner of the object commits to a mechanism to sell to the n^t buyers. A selling mechanism for n buyers, M, is a triple, (B, ξ, τ), where $B = \times_{i=1}^{n} B_i$ is a profile of arbitrary finite message spaces (B_i being i's space), $\xi : B \to \{(x_0, x_1, \ldots, x_n) \in [0,1]^{n+1} | \sum_i x_i = 1\}$ is a rule mapping a vector of messages ("bids") to a (possibly random) allocation of the object, and $\tau : B \to [0, v_H]$ is a rule determining expected payments of buyers as a function of the messages.[4] Let τ_i and ξ_i be i-th element of τ and ξ that corresponds to the payment and allocation rule for bidder i, respectively. ξ_0 then stands for the probability that the object remains with the seller. The seller cannot force participation in the auction. Therefore, we require B_i for all i to include a non-participation option, $\emptyset$, which, when exercised, results in no winning and no payment for bidder i. Letting $b = (b_1, \ldots, b_n)$ denote a profile of n^t bids, we require $\xi_i(\emptyset, b_{-i}) = \tau_i(\emptyset, b_{-i}) = 0$. Finally, for each $n \in \mathbb{N}$ let $\mathcal{M}(n)$ be an arbitrary set of mechanisms for n buyers that contains at least all direct mechanisms.

Upon observing the selling mechanism, the buyers simultaneously place their bids. To streamline the analysis we assume that n^t is known to all traders before the mechanism is played in t, and that the new traders in n^t do not observe any of the previous history.

Once all bids are received, the outcome of the mechanism is implemented. The object is allocated to one of the n^t buyers or remains with the seller and payments are made. If the object remains with the seller, she consumes it and the game ends.[5] Instead, if the object goes to one of the n^t buyers, the game continues into a new round $t + 1$ of trade and the player who is awarded the object is identified as seller, player 0, in round $t + 1$. To minimize irrelevant notation we assume that the winning bidder does not observe the bids of the competing buyers.

Payoffs. Traders maximize their discounted net surplus. Payoffs are as follows. Let $\delta \in (0, 1)$ be the common discount factor. To avoid trivialities, we require $\delta v_H > v_L > 0$. Denote with $M^t = (B^t, \xi^t, \tau^t)$ the mechanism chosen in period t and with b^t the profile of bids. Let v_i^t be the

[4] As we explain later, imposing any finite bound on the maximum payment avoids equilibrium bubbles.

[5] Note that the seller can always set up a mechanism where the object remains with her and consumption takes place with probability one.

value of trader i in period t. Now, suppose a trader is a new buyer i in period $t-1$ and let $S^t(i) = 1$ if i becomes seller in t and otherwise $S^t(i) = 0$.[6] Abusing notation, we will refer to this trader as i in period $t-1$ and as 0 in period t if he is the seller in t. The end-of-game payoff of i following the specified history is:

$$-\tau_i^{t-1}(b^{t-1})\delta^{t-1} + \delta^t S^t(i) \left(\sum_{j=1}^{n^t} \tau_j^t(b^t) + \xi_0^t(b^t) v_0^t \right),$$

where $\tau_i^t(b^{t-1})$ is the payment made by i in round $t-1$, and $S^t(i) \sum_{j=1}^{n^t} \tau_j^t(b^t)$ is the payments i collects in round t and $S^t(i)\xi_0^t(b^t)v_0^t$ is the t-period payoff i makes if the object remains unsold in t and therefore consumption follows.

Equilibrium. Let $\mathcal{M} = \cup \mathcal{M}(n)$ and let $\mathcal{B} = \cup_{M \in \mathcal{M}} B(M)$ be the union of all message spaces over all admissible mechanisms. The behavioral strategy of each new trader is composed by (i) a behavioral bidding strategy, $\beta_i : \{v_L, v_H\} \times \mathbb{N} \times \mathcal{M} \to \mathcal{B}$ such that $\beta_i^t(\cdot, M) \in B(M)$ for all $M \in \mathcal{M}$, mapping the valuation, the realized number of buyers and the selling mechanism into a, possibly random, bid and (ii), relevant at each information set in which the buyer becomes an owner in the subsequent period, a mechanism selection strategy $\mu : \mathcal{M} \times \mathcal{B} \times \mathbb{N} \times \{v_L, v_H\} \to \Delta(\mathcal{M})$ with the support of $\mu(\cdot, n)$ in $\mathcal{M}(n)$ for all n, that maps what the buyer has observed in playing the previous round (i.e., the mechanism and his bid), his value and the number of bidders into a, possibly random, mechanism that he posts.[7] As standard, a perfect Bayesian equilibrium is a profile of strategies, such that, for all traders, the expected continuation payoff from choosing a different strategy is, at any information set, lower than the equilibrium payoff.

When we discuss uniqueness of equilibrium payoffs, we restrict attention to a specific class of equilibrium strategies. In particular, we will let the strategy of sellers only depend on their valuation and the number of their buyers, but not on payoff irrelevant previous history. Moreover, we will look for a symmetric equilibrium, where all buyers adopt the same strategy when reselling. We claim that these requirements are without loss of generality, but simplify the proofs considerably.

[6] S does not only depend on the mechanism posted in period t, M^t, and on the bids placed, b^t, but also on the realization of nature's move in case the allocation rule prescribes a lottery among several agents.

[7] To avoid unnecessary complications we assume that a seller who face n buyers is restricted to pick a mechanism within the $\mathcal{M}(n)$ class. Also, observe that the strategies defined above are unnecessarily general. It simplifies notation.

3 Equilibrium of the network trading game

We characterize the unique equilibrium outcome of this trading game. Our first observation determines the equilibrium payoff of a high value seller at an arbitrary period.

Lemma 1 *In any equilibrium, a high-value seller at period $t \geq 0$ proposes a mechanism where he retains the object and consumes it with probability one.*

We now provide a sketch of the proof. The formal proof is just a rewriting of this argument and is therefore omitted. By consuming at the end of period t the seller can secure v_H, whereas the maximum profit he can make from selling is δv_H since this is the maximum surplus attainable in the economy if the object is not consumed and brought forward for another round of trade.[8]

We now determine the equilibrium payoff of a low-value seller at the beginning of an arbitrary period. We do this in two steps. First, we fix the number of buyers drawn by the seller and their continuation payoffs, and we derive the unique equilibrium payoff of the seller in this static game. In the second step, we endogenize the buyers' continuation payoff and show that it is unique and consistent with the optimal trading mechanism we obtained in the first step.

First step. Consider a low-value seller with n buyers. The mechanism that the seller will offer depends on buyer's willingness to pay for the object. From Lemma 1 we know that if a high-value buyer acquires the object, he will consume it in the next period thereby obtaining a continuation payoff of v_H. Hence, a high-value buyer willingness to pay is δv_H.

A low-value buyer who acquires the object has the opportunity to re-sell it to his buyers and consume it in case of no-sale. Hence, his willingness to pay is larger than δv_L, but smaller than δv_H. We denote by V the continuation payoffs that a low-value buyer obtains from the subgame starting at subsequent period that ensues upon acquiring the object. The willingness to pay of a low-value buyer is therefore δV. Since we focus on equilibrium sustained by a symmetric strategy profile we have that all low-value buyers will have the same willingness to pay δV.

[8]The assumption that the maximum payment is bounded implies that, if a seller obtains payoff higher than δv_H from selling, then at least one other trader downstream must obtain a payoff below zero. However, this is impossible in equilibrium in the current setup as buyers can all guarantee themselves a payoff of zero by exercising their outside option. If there was no bound in maximum payments, one could sustain a bubble-equilibrium where agents exchange the object forever, at price v_H/δ^t in each period t.

Given these considerations, we can study the low-value seller's choice of the selling mechanism in the static-game in any given period as a mechanism design problem where each of the n buyers have two possible willingness to pay for the object: δV with probability $1-\mu$ and δv_H with probability μ. We therefore construct equilibria where low-value sellers select the incentive compatible and individually rational direct selling mechanism that maximizes their revenue and all buyers report truthfully their valuations.[9]

At this purpose, let V^* be defined as follow

$$\delta V^* = (1-\mu)v_L + \mu\delta v_H.$$

The optimal selling mechanism depends on the relation between V and V^*. Since this is a very well known result, we will formally state the payoffs only, and discuss informally the mechanisms used.

If $V < V^*$ an optimal incentive compatible direct mechanism for the seller is as follows:

a) The seller asks bidders to declare whether they have high or low value;

b) The object is sold at price δv_H, to a random buyer among those that reported high value, if any did;

c) The seller consumes the object at the end of period if no buyer reports a high-value.

We refer to this mechanism as *exclusive posted price*, because it excludes low-value buyers from the sale.[10]

On the other hand, if $V \geq V^*$, the seller runs a mechanism akin to a second-price auction:

a) The sellers asks bidders to declare whether they have high or low value;

b) The object is sold at price δV to a random buyer if all buyers declare low value;

c) It is sold at price δv_H to one randomly selected buyer among those that reported high-value, if at least two buyers declare high-value;

[9] Restricting attention to incentive compatible and individually rational direct mechanisms is without loss of generality. Since buyers can opt out, individual rationality ensures that every type of every bidder obtains a payoff weakly larger than zero.

[10] It is obvious that this mechanism is individually rational and buyers have an incentive to report truthfully their valuation.

d) If only one buyer reports having high-value, the seller sells to this buyer at a price $\tilde{p}$ that solves:[11]

$$\delta v_H - \tilde{p} = \frac{1}{n}\left[\delta v_H - \delta V\right].$$

We refer to this mechanism as *inclusive auction*, because trade always takes place, even if all buyers report a low-valuation.

Lemma 6.1.1 in the Appendix formally defines these two mechanisms and show that they are optimal and incentive compatible. Here we only report the expected payoff that a low-value seller with n buyers obtains. This is:

$$R(n,V) = \begin{cases} \delta v_H[1-(1-\mu)^n] + v_L(1-\mu)^n & \textit{if } V < V^* \\ \delta v_H[1-(1-\mu)^{n-1}] + \delta V(1-\mu)^{n-1} & \textit{if } V \geq V^* \end{cases} \tag{1}$$

Observe that, as long as all buyers have continuation payoff equal to V, then this payoff is uniquely determined. The fact that, in any equilibrium, all buyers have the same V is essential for proving uniqueness of equilibrium payoffs and follows from our assumptions on equilibrium strategies.

Second step. To complete the equilibrium characterization, we need to pin down the continuation payoffs of low-value sellers V, which is endogenously determined by equilibrium play. This continuation payoff V must be consistent with equilibrium in the static game, which we described earlier (see Lemma 1 here and Lemma 6.1.1 in the appendix).

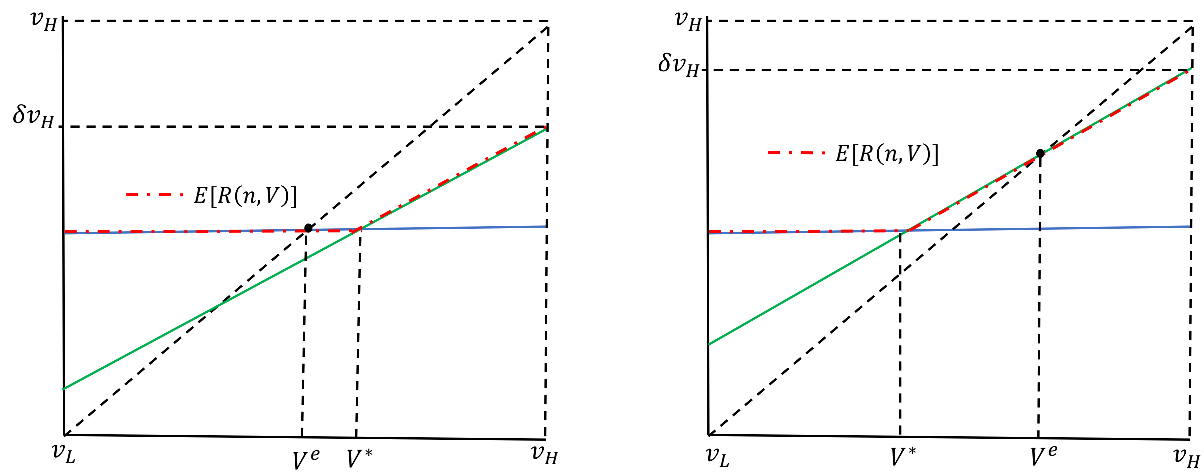
In particular, since V is the expected selling profit for a low-value seller at the beginning of an arbitrary period t prior to the realization of the number of buyers n, we have that the equilibrium value of V, denoted by V^e, solves:

$$V^e = \mathbb{E}[R(n,V^e)] = \sum_{n=0}^{\infty} \pi_n R(n,V^e) \tag{2}$$

where the revenue function $R(\cdot,\cdot)$ is defined in (1).

Finding all solutions to the above equation completes the heuristic equilibrium characterization. It turns out that a solution exists and it is unique. $\mathbb{E}[R(n,V)]$ is a continuous function mapping $[v_L, v_H]$ into $[v_L, \delta v_H]$ and so existence follows from Brouwer's fixed point Theorem. Uniqueness follows since $\mathbb{E}[R(n,V)]$ is increasing with slope less than one.

[11] The price $\tilde{p}$ is such that a high-value buyer is indifferent between reporting being a high type and reporting being a low type, given that all other buyers report their type truthfully.



(a) Posted price equilibrium

(b) Auction equilibrium

Figure 1: Equilibrium low-value buyer continuation payoff V^e

Figure 1 illustrates this argument geometrically. In Figure 1 the independent variable V is on the x-axis and the dotted red line represents the expected revenue of a low-value seller, $\mathbb{E}[R(n, V)]$. Note that for $V < V^*$ the payoff of the seller is maximized through an exclusive posted price, and so the revenue is constant and independent of V (blue line). Instead, for $V > V^*$, the payoff of the seller is maximized through an inclusive auction, in which case the revenue is a linear function of V, with slope less than one (green line). With this in mind, an equilibrium is found where the 45-degree line crosses $\mathbb{E}[R(n, V)]$.

Figure 1.a shows an equilibrium where the seller uses an exclusive posted price. This is the case whenever the point V^* is to the left of the point of intersection between the 45-degree line and the revenue from exclusive posted prices. When V^* is on the right of that point, as in Figure 1.b, we have an equilibrium where the seller uses an inclusive auction. Formally, the equilibrium will exhibit inclusive auctions when:

$$V^* = (1-\mu)v_L/\delta + \mu v_H \leq \sum_n \pi_n[\delta v_H[1-(1-\mu)^n] + (1-\mu)^n v_L] \ \ or$$

$$T(\pi, \delta) \equiv \delta \frac{\delta - \mu - \delta \sum_n \pi_n (1-\mu)^n}{1 - \mu - \delta \sum_n \pi_n (1-\mu)^n} - \frac{v_L}{v_H} \geq 0.$$

Otherwise, when $T(\pi, \delta) < 0$, in equilibrium sellers will use an exclusive posted price mechanism.[12]

[12]When $T(\pi, \delta) = 0$, both equilibria coexist as they provide they seller with the same payoff. Since it does

With the above discussion in mind, the following proposition characterizes equilibrium payoff of the seller. Before proceeding we introduce a last key piece of notation. Let

$$L(\pi) \equiv \sum_n \pi_n (1-\mu)^n = \mathbb{E}[(1-\mu)^n]$$

denote the prior probability that all buyers will have low value.[13]

Proposition 1 *An equilibrium exists and, in all equilibria, the continuation payoff and the strategy of the initial low-value seller is as follows:*

i. If $T(\pi, \delta) < 0$ then the initial low-value seller uses exclusive posted prices and her expected payoff is

$$V^e(\pi, \delta) = \delta v_H (1 - L(\pi)) + v_L L(\pi) \tag{3}$$

ii. If $T(\pi, \delta) \geq 0$ then then the initial low-value seller uses inclusive auctions and her expected payoff is

$$V^e(\pi, \delta) = \delta v_H \frac{1 - L(\pi)/(1-\mu)}{1 - \delta L(\pi)/(1-\mu)}$$

Proposition 1 is stated without proof. It is a direct implication of our equilibrium characterization Theorem 1, which is stated and proved in the Appendix.

Except from the initial seller, all other traders are ex-ante identical. We refer to traders except the initial seller as to dealers. When $T(\pi, \delta) < 0$ the initial seller uses an exclusive posted price and so either trade stops in period 0 with the seller consuming the object or the object is bought by a high-value buyer who consumes in period 1. The equilibrium total surplus therefore coincides with the initial seller's expected payoff, whereas other traders make no profit.

When $T(\pi, \delta) \geq 0$ the initial seller, and each low value seller along the play of the game, uses an inclusive auction, and so the object is bought and resold amongst dealers until a high value dealer acquires the object and consumes. Because payments are welfare neutral in this model, we can write the total surplus (collected by all traders) evaluated at period zero as satisfying the following

not, generically, restrict the set equilibrium payoffs, we will henceforth assume that the seller uses auctions when indifferent.

[13]For given $0 \leq \lambda \leq 1$, the function $\mathbb{E}[\lambda^x]$ is also called the *probability generating function* of the random variable x. Probability generating functions are easily computable for various discrete distributions.

recursive equation

$$W(\pi, \delta) = \underbrace{(1 - L(\pi))}_{\substack{\text{Prob. of finding} \\ \text{a high-value}}} \delta v_H + \underbrace{L(\pi)}_{\substack{\text{Prob. all buyers} \\ \text{are low-value}}} \delta W(\pi, \delta)$$

The solution to this equation is stated in the ensuing proposition , which also reports the equilibrium total dealers' surplus from Theorem 1 (in the Appendix), which we denote U^e. Again, further proof is omitted.

Proposition 2 *If $T(\pi, \delta) \geq 0$ equilibrium total surplus is*

$$W^e(\pi, \delta) = \delta v_H \frac{1 - L(\pi)}{1 - \delta L(\pi)} \tag{4}$$

and the equilibrium total dealers' surplus is

$$U^e(\pi, \delta) = W^e(\pi, \delta) - V^e(\pi, \delta) = \delta v_H \left[\frac{1 - L(\pi)}{1 - \delta L(\pi)} - \frac{1 - L(\pi)/(1 - \mu)}{1 - \delta L(\pi)/(1 - \mu)} \right]$$

The dealer surplus is obtained by taking the difference between equilibrium total surplus and initial seller's expected payoff. It reflects the information rents that the initial seller leaves to dealers.

3.1 Network structure and payoffs

We now show how differences in the network structure, as summarized by π, affect the equilibrium total surplus and its distribution between the initial seller and the dealers. Remarkably, the network affects equilibrium outcomes exclusively through $L(\pi)$ which, recall, is the probability that all buyers have low-value.

Proposition 3 *Consider π and π', with*

$$L(\pi) < L(\pi').$$

Then:

(i) Equilibrium total surplus and initial seller's expected payoff are higher in π than π';

(ii) Total dealers' expected surplus is lower in π than in π' if $T(\pi', \delta) > 0$, it is higher if $T(\pi', \delta) < 0 \leq T(\pi, \delta)$ and constant otherwise.

The proof of the proposition is in the Appendix. We sketch here the main arguments. When $L(\pi) < L(\pi')$ the probability that a seller meets a high type buyer is higher under π than under π'. If the initial seller employs the posted price mechanism, this means that under π she is more likely to sell at δv_H than under π'. Total surplus and initial seller's profit are both higher.

If sellers use inclusive auction, then under π a high value buyer will be found, in expectation, after fewer rounds of resale. Hence, total equilibrium surplus will be higher in network π. The reason that the initial seller is also better off under π is because information rents are lower under π than π'. The intuition is the following. The incentive of a high value buyer to misreport his type comes from the prospect that, in the event all other buyers have a low value, he has a positive probability to receive the object at a price δV. The probability that this happens is lower under π relative to π', and so the seller has to leave less information rent on the table.

Turning to equilibrium total dealers' surplus, the proposition states that equilibrium surplus of buyers is weakly lower under π than under π', unless the seller is using posted prices under π but uses auctions instead under π', that is if $T(\pi', \delta) < 0 \leq T(\pi, \delta)$.[14]

Using this result, we can now perform some typical comparative statics about changes in the network structure. We consider first-order stochastic shifts and mean preserving spreads in the degree distribution.[15]

Corollary 1 *Suppose $\pi \neq \pi'$ and π' first-order stochastically dominates π. Then:*

(i) Equilibrium total surplus and initial seller's expected payoff are higher in π' than π;

(ii) Total dealers' expected surplus is lower in π' than in π if $T(\pi, \delta) > 0$, it is higher if $T(\pi, \delta) < 0 \leq T(\pi', \delta)$ and constant otherwise.

Suppose $\pi \neq \pi'$ and π' is a mean-preserving spread of π. Then:

(i) Equilibrium total surplus and initial seller's expected payoff are higher in π than π';

(ii) Total dealers' expected surplus is lower in π than in π' if $T(\pi', \delta) > 0$, it is higher if $T(\pi', \delta) < 0 < T(\pi, \delta)$ and constant otherwise.

[14]It is easy to verify that $T(\pi', \delta) < T(\pi, \delta)$.

[15]We say that π' is a first-order stochastic shift from π if $\sum_{n=1}^{k} \pi_n \leq \sum_{n=1}^{k} \pi'_n$ for all integer $k > 0$. Instead, π' is a mean-preserving spread of π if $\sum_n n\pi_n = \sum_n n\pi'_n$ and $\sum_{k=1}^{j} \sum_{n=1}^{k} \pi_n \leq \sum_{k=1}^{j} \sum_{n=1}^{k} \pi'_n$ for all integer $j > 0$.

Proof Observe that $(1-\mu)^n$ is decreasing and convex in n. Then, it follows from Proposition 6.D.1 from Mas-Colell, Whinston and Green (1995) that if π' first-order stochastically dominates π then $\sum_n \pi'_n(1-\mu)^n \leq \sum_n \pi_n(1-\mu)^n$. It follows from Proposition 6.D.2. from Mas-Colell, Whinston and Green (1995) that if π' is a mean-preserving spread of π, then $\sum_n \pi'_n(1-\mu)^n \geq \sum_n \pi_n(1-\mu)^n$. The corollary then follows from Proposition 3. □

In words, an increase in the density of the network of resale (first-order-stochastic shift in the distribution of connection), always increases total equilibrium surplus and initial seller's equilibrium profits. In contrast, an increase in the dispersion of the network of resale, summarized by a mean-preserving-stochastic shifts in the distribution of connections, always decreases total equilibrium surplus and initial seller's equilibrium profits. Whether dealers benefit from denser or more dispersed networks depend also on whether the change in the network affects the choice of the optimal mechanism. In the region of parameters where sellers use inclusive auctions both before and after the change, dealers loose from first order stochastic shifts in the network, but gains from mean preserving spread shifts.

4 Efficient outcomes

Next, we determine what a central planner, solely interested in the maximization of total surplus, can achieve if she could select, ex-ante, the strategy of all types of traders. Equivalently, we are interested to the equilibrium outcome of a model were all players share the common objective of maximizing expected total surplus. Using this benchmark the distortions due to incentive problems are abstracted away. Nonetheless, relative to first-best, it exhibits a loss since the planner lacks knowledge of he realized network structure and valuations.[16]

We first note that, from the planner's perspective, sellers' strategies must satisfy two immediate requirements. First, high-value sellers must always consume. Second, low-value sellers must transfer the object to a high-value buyer, whenever there is one. Note that the equilibrium strategies that we have derived in the previous section satisfy these two requirements.

[16] At one extreme, one could look at an omniscient central planner. In this case the object would go straight to the high-value agent which is closest to the initial seller. The ex-ante payoff W would in this case be equal to $v_H \frac{\delta \sum_n \pi_n(1-(1-\mu)^n)}{1-\delta \sum_n \pi_n n(1-\mu)^n}$.

Hence, the potential difference between efficient and equilibrium behavior arises about whether, when facing only low-value buyers, the low-value seller should consume or instead pass on the object to one of the low-value buyers, who will then try to resell it to find a high-value buyer.

Under the policy of transferring the object to a low-value buyer, the expected total surplus coincides with the surplus we computed in 4. The alternative policy is to only pass over the object if there is a high-value buyer. This is precisely the welfare level achieved under the posted-price mechanism, which is computed in equation 3.

We conclude that a low-value seller will (welfare optimally) transfer the object to a low-value buyer if and only if:

$$T^{fb}(\pi, \delta) \equiv \delta^2 \frac{1 - L(\pi)}{1 - \delta L(\pi)} - \frac{v_L}{v_H} \geq 0.$$

Some algebra then shows that

$$T^{fb}(\pi, \delta) > T(\pi, \delta).$$

It follows that equilibrium might be inefficient only when sellers use posted prices, if the welfare optimal policy is to use auctions instead.[17]

Remark 1 *The equilibrium outcome is inefficient if and only if* $T(\pi, \delta) > 0 > T^{fb}(\pi, \delta)$.

In equilibrium there might be too much consumption by low-value sellers compared to what would be optimal from a welfare perspective. The intuition for this result is that a low value seller does not internalize all the positive externalities realized when the object is allocated to a low value trader and resold in the subsequent periods. This lack of internalization is a consequence of asymmetric information: when the seller adopts the inclusive auction he must leave information rent to high value traders, and so the seller only gets a share of the surplus generated by reselling.

However efficiency is always attained by equilibrium when traders are sufficiently patient, regardless of the resale network. In particular, it is immediate to see that

$$\lim_{\delta \to 1} T(\pi, \delta) = 1$$

which, together with the previous remark, implies the following result.

[17] $\delta \frac{\delta - \delta L(\pi) - \mu}{1 - \delta L(\pi) - \mu} < \delta \frac{\delta - \delta L(\pi)}{1 - \delta L(\pi)}$ follows from Lemma 2, which is stated and proved in the Appendix, using $a = \delta$, $b = 1$, $c = \delta L(\pi) - \mu$, $d = \delta L(\pi)$.

Remark 2 *There always exists δ large enough such that sellers use auctions and the equilibrium outcome is efficient.*

5 Conclusions

We put forward a model of dealer networks with the following three key assumptions: (i) there is wide-spread uncertainty about the web of downstream resale opportunities and the network is summarized by a degree distribution common to all dealers, (ii) it's a seller's market, where full bargaining power belongs to the current owner of the object and (iii) asymmetric information, about the private value of selling to a captive client, is the determinant of the bargaining surplus of dealers. Despite being a very stylized model, it captures two central aspects of dealer networks. First, payoffs are determined by rents that dealers obtain by having private information about resale opportunities. Second, due to resale, the willingness to pay of dealers is endogenous and interdependent with that of other dealers, through the network architecture.

The simple equilibrium characterization allows us to perform comparative statics on globally changing the network structure. In our model, the network affects payoffs through its effect on a specific point of the probability generating function of the degree distribution — that is the probability, computed before the number of connections becomes known, that all potential downstream connection will have a low private value.

The two main features of our trading protocol (i.e., bargaining under asymmetric information and seller's bargaining power) are certainly essential to our results. Future research is needed to understand the impact of relaxing our assumption on to the information that dealers have about the network. In particular, we consider the current model as a benchmark in a larger family of models where dealers have different information about the network structure. A natural first step, then, would be to assume that dealers have private information about the number of downstream connections, when negotiating with an upstream seller. In this case, the endogenous continuation value of low-value dealers would also depend on the number of connections and private information about the network would provide additional information rent. This is the subject of ongoing work.

6 Appendix

6.1 Existence, uniqueness and payoffs

This section proves existence and uniqueness. We first record well-known results about optimal mechanisms in 6.1.1, which serve as building blocks for our construction of an equilibrium in symmetric and stationary strategies. We then prove uniqueness of payoffs.

6.1.1 Optimal mechanisms

Consider a (sub)game where a seller, with valuation v_L or v_H, face n buyers. A buyer's valuation is either δV with probability μ or δv_H with probability $1-\mu$, where $v_L < V < v_H$, $v_L < \delta v_H$, and $0 < \mu < 1$. We now recall well-known results about optimal mechanisms.

Let $B_i^* = \{\emptyset, \delta V, \delta v_H\}$ for all $i = 1, \ldots, n$ and $B^* = \times_i B_i^*$ (i.e., bidders can only send one of three messages, either their continuation value from buying or the opt-out message). For all $b \in B^*$, let $\xi_0^*(b) = 1 - \sum_{i \neq 0} \xi_i^*(b)$ and write $\#H(b)$, resp., $\#L(b)$, for the number of reports δv_H, resp., δV, in the profile b.

The following mechanism $M^*(v_L, n) = (B^*, \xi^*, \tau^*)$ is an optimal mechanism for the **low-value seller**:

A.1.) If $V < V^*$ the seller employs an exclusive posted-price mechanism. That is, for all $i \neq 0$, $b \in B$:

$$\xi_i^*(b_i, b_{-i}) = \begin{cases} 0 & \textit{if } b_i \in \{\emptyset, \delta V\} \\ \frac{1}{\#H(b)} & \textit{if } b_i = \delta v_H \end{cases} \qquad \tau_i^*(b_i, b_{-i}) = \begin{cases} 0 & \textit{if } b_i \in \{\emptyset, \delta V\} \\ \frac{\delta v_H}{\#H(b)} & \textit{if } b_i = \delta v_H \end{cases}$$

(i.e., the object is randomly allocated among those buyers who report high-value, who all pay an equal share of the reported value. If no buyer report high-value, the seller retains the object and nobody pays anything.)

A.2.) If $V \geq V^*$ the seller employs an inclusive auction mechanism. That is, for all $i \neq 0$, $b \in B$:

$$\xi_i^*(b) = \begin{cases} 0 & \textit{if } b_i = \emptyset \\ 0 & \textit{if } b_i = \delta V, \#H(b) > 0 \\ \frac{1}{\#L(b)} & \textit{if } b_i = \delta V, \#H(b) = 0 \\ \frac{1}{\#H(b)} & \textit{if } b_i = \delta v_H \end{cases}$$

$$\tau_i^*(b) = \begin{cases} 0 & \textit{if } b_i = \emptyset \\ 0 & \textit{if } b_i = \delta V, \#H(b) > 0 \\ \frac{\delta V}{\#L(b)} & \textit{if } b_i = \delta V, \#H(b) = 0 \\ \frac{\delta[v_H(\#L(b)-1)+V]}{\#L(b)} & \textit{if } b_i = \delta v_H, \#H(b) = 1 \\ \frac{\delta v_H}{\#H(b)} & \textit{if } b_i = \delta v_H, \#H(b) > 1 \end{cases}$$

(i.e., the object is randomly allocated among all buyers reporting high-value, if any; otherwise, unless all agents opt-out, it is randomly allocated among all buyers who report low-value. Except if a single buyer reports high-value, all agents pay a fraction of their reported value equal to the probability with which they obtain the object. If there is a single buyer reporting high-value, then he pays an amount that makes him different obtaining the object for sure at that amount, and what he would expect otherwise by reporting a low-valuation. If all buyers opt-out, then the seller retains the object and nobody pays anything.)

The following mechanism $M^*(v_H, n) = (B^*, \xi^*, \tau^*)$ is optimal for the **high-value seller**:
A.3) The seller always consumes the object. That is, for all $i \neq 0$, $b \in B^*$ set:

$$\xi_i^*(b_i, b_{-i}) = \tau_i^*(b_i, b_{-i}) = 0.$$

(B) Let M^* be any of the mechanisms defined in (A.1),(A.2) and (A.3). These mechanism are incentive compatible and individually rational. That is, the following $b^*(v_i, n, M^*)$ is an equilibrium bidding strategy for **all buyers** for M^*. For all $i \neq 0$ let:

$$b_i^*(v_i, n, M^*) = \begin{cases} \delta V \;\; \textit{if } v_i = v_L \\ \delta v_H \;\; \textit{if } v_i = v_H \end{cases}$$

(C) The payoffs are as follows.
The **high-value seller** achieves a payoff of v_H.

The **low-value seller** achieves an expected payoff, denoted $R(n, V)$, equal to:

$$R(n, V) = \begin{cases} \delta v_H[1 - (1-\mu)^n] + v_L(1-\mu)^n & \textit{if } V < V^* \\ \delta v_H[1 - (1-\mu)^{n-1}] + \delta V(1-\mu)^{n-1} & \textit{if } V \geq V^* \end{cases}$$

The payoff of a **low-value buyer** is 0.

The payoff of an **high-value buyer** is:

$$U(n,V) = \begin{cases} 0 & \textit{if } V < V^*, \\ \delta(V_H - V)(1-\mu)^{n-1}/n & \textit{if } V \geq V^*. \end{cases}$$

6.1.2 Existence and equilibrium payoffs

We now use the above result to construct an equilibrium and thus prove existence. We now describe the strategy profile. The description is rather informal; a formal description would be cumbersome and tedious without adding much. First, assume that $T(\pi,\delta) < 0$ and let $V := \delta v_H(1 - L(\pi)) + v_L L(\pi)$.

i. Upon acquiring the object, a high-value seller offers the mechanism $M^*(v_H, n)$ when he faces n buyers.

ii. Upon acquiring the object, a low-value seller select the posted mechanism $M^*(v_L, n)$ when he faces n buyers.

iii. At histories where the mechanism is M^*, bidder i follows the bidding strategy $b_i^*(n, v_i)$ when his valuation is v_i.

iv. At histories where the mechanism is $M(n) = (B, \xi, \tau)$, different from $M^*(v_L, n)$ and $M^*(v_H, n)$, consider the static game, where the set of actions of bidder i is B_i and bidder i's payoff is $-\tau_i(b_i, b_{-i}) + \xi_i(b_i, b_{-i})\delta v_i$ when the profile of bids is (b_i, b_{-i}), with v the type of bidder i, $v_i \in \{V, v_i\}$. Since this is a finite game of incomplete information, there exists an equilibrium (b_i^*, b_{-i}^*). We let bidder i play $b_i^*(V)$ (resp., $b_i^*(v_H)$) when his valuation is V (resp., v_H)

This completely describes the strategies. Note that there are symmetric and stationary. We do not describe the beliefs. We only need that upon observing a mechanism other than $M^*(v_L, n)$ and $M^*(v_H, n)$, a bidder continues to assume that the distribution of valuations of current and subsequent bidders remains $(\mu, 1-\mu)$, i.e., it is a condition of "don't signal what you don't know."

We now argue that this strategy profile is (part of) a perfect Bayesian equilibrium. First of all, it is immediate to verify that upon acquiring the object, a high-value seller obtains a payoff of v_L along the equilibrium path. Similarly, the payoff of a low-value seller is V since he uses a posted price. Second, since $T(\pi, \delta) < 0$ is equivalent to $V < V^*$, the posted price mechanism is optimal within the set of incentive compatible and individually rational direct mechanisms. Third, consider any seller i's private history $((B, \xi, \tau), n, b_i, n')$ where seller i has acquired the object at the previous period and now faces a market with n' buyers. The strategy specifies that seller i offer the mechanism $M^*(v_L, n)$ (resp., $M^*(v_H, n)$) if his valuation is v_L (resp., v_H). If the seller offers the mechanism (B', ξ', τ') instead, his expected payoff coincides with the one he would obtain in a static mechanism design problem, if he were to offer the mechanism (B', ξ', τ') when facing n' buyers, each with valuation either V or v_H. By the revelation principle, the seller's revenue is lower than the maximal revenue we can obtain over all incentive compatible and individually rational direct mechanisms. Therefore, the deviation cannot be profitable. Fourth, consider any buyer i's private history $((B, \xi, \tau), n)$, i.e., the buyer faces $n - 1$ other buyers and the selling mechanism is (B, ξ, τ). From our previous observation, upon acquiring the good, the buyer's continuation payoff is V (resp., v_H) if his valuation is v_L. If he does not acquire the good, his continuation payoff is zero. By construction, the buyer has no one shot profitable deviation. It remains to argue that the buyer has no profitable deviation. The argument is nearly identical to the the proof of the one-shot deviation principle, see Hendon et al, GEB, 1996. To the contrary, suppose that the buyer has a profitable deviation $(b_i, (M(n))_{n\in\mathbb{N}})$, which results in the expected payoff $-\tau_i(b_i, b^*_{-i}) + \xi_i(b_i, b^*_{-i})\delta V'$. From our previous argument, it is lower than $-\tau_i(b_i, b^*_{-i}) + \xi_i(b_i, b^*_{-i})\delta V$ and, therefore, lower than $-\tau_i(b^*_i, b^*_{-i}) + \xi_i(b^*_i, b^*_{-i})\delta V$. This completes the proof.

We now assume that $T(\pi, \delta) \geq 0$ and let $V := \delta v_H \frac{1-L(\pi)/1-\mu}{1-\delta L(\pi)/1-\mu}$. The only difference with the previous case is that the low-value seller offers the auction mechanism $M^*(v_L, n)$ when he faces n buyers. We only have to argue that it is indeed optimal for the low-value seller to offer the auction mechanism. By construction, the expected continuation payoff of a bidder winning the auction is either V or v_H. Therefore, the auction mechanism is optimal if $V \geq V^*$, which is true by construction.

Theorem 1 *The end-of-game equilibrium payoffs are:*

v. The payoff of the initial low-value seller in period 0 is:

$$V^e(\pi,\delta) = \begin{cases} \delta v_H - (\delta v_H - v_L)\sum_n \pi_n(1-\mu)^n & \text{if } T(\pi,\delta) < 0, \\ \delta v_H \frac{1-\sum_n \pi_n(1-\mu)^{n-1}}{1-\delta\sum_n \pi_n(1-\mu)^{n-1}} & \text{if } T(\pi,\delta) \geq 0. \end{cases}$$

vi. Low-value buyers obtain a payoff of zero.

vii. High-value buyer i in period t expects to make:

$$U_i^e(\pi,\delta) = \delta^t \mathbb{E}[U(n, V^e)] = \begin{cases} 0 & \text{if } T(\pi,\delta) < 0, \\ \delta^{t+1}(v_H - V)\sum_n [\pi_n(1-\mu)^{n-1}/n] & \text{if } T(\pi,\delta) \geq 0. \end{cases}$$

viii. The total surplus generated in the economy is:

$$W^e(\pi,\delta) = \begin{cases} \delta v_H(1-\sum_n \pi_n(1-\mu)^n) + \sum_n \pi_n(1-\mu)^n v_L & \text{if } T(\pi,\delta) < 0, \\ \delta v_H(1-\sum_n \pi_n(1-\mu)^n)/(1-\delta) & \text{if } T(\pi,\delta) \geq 0. \end{cases}$$

6.1.3 Uniqueness of payoffs

Fix any symmetric and stationary equilibrium (β,μ). Consider two period-t histories $((B,\xi,\tau),n,b_i)$ and $((B',\xi',\tau'),n',b_i')$ at which bidder i obtains the object and let V_i and V_i' be bidder i's expected continuation payoff under (β,μ) when bidder i's valuation is v_L. If bidder i's valuation is v_L, assume by contradiction that $V_i < V_i'$. Bidder i can deviate at the history $((B,\xi,\tau),n,b_i)$ and offers the mechanism he would offer at history $((B',\xi',\tau'),n',b_i')$.[18] Since the bidders active at period $t+1$ do not observe the mechanism and bids at period t, bidder i's expected continuation payoff at history $((B,\xi,\tau),n,b_i)$ is $V_i' > V_i$, a profitable deviation. Therefore, $V_i = V_i'$. In words, regardless of the mechanisms and bids, a successful bidder must have a unique continuation payoff V_i. Similarly, if bidder i's valuation is v_H, his expected continuation payoff is v_H.

By symmetry, any two successful bidders must have the same expected continuation payoff V upon acquiring the object, when their valuation is v_L.

By stationarity, if bidder i owns the object at period t (and, therefore, has acquired it at period $t-1$), his expected payoff must be V when his valuation is v_L. In addition, $V = \sum \pi_n V_n$, where

[18] More precisely, we consider an alternative strategy for bidder i where at history $(B,\xi,\tau),n,b_i)$ the new strategy specifies the same distribution over mechanisms than strategy μ_i^t at history $(B',\xi',\tau'),n',b_i')$.

V_n is the expected revenue to bidder i when facing n bidders, who either value the object at δV or δv_H with probability $1-\mu$ and μ, respectively.

Finally, it must be the case that V_n is the maximal revenue bidder i can achieve. To prove that claim, we construct mechanisms, which approximate the maximal revenue derived in 6.1 in all equilibria. Therefore, bidder i can always approximate the maximal revenue by offering one of these mechanisms. Since the approximation can be made arbitrary, it must be that V_n is the maximal revenue bidder i can achieve. We now turn to the construction of these mechanisms.

Choose $\varepsilon > 0$ such that $\delta V < \delta v_H - \varepsilon$. We first modify the exclusive posted mechanism as follows: if at least one bidder bids δv_H, the successful bidder pays $(\delta v_H - \varepsilon)/\#H(b)$. We now verify that there exists a unique equilibrium payoff of this modified mechanism. Throughout, we fix an equilibrium (β_i, β_{-i}) and write "proba(E)" for the probability of the event $E \subseteq \{\delta V, \delta v_H, \emptyset\}^{n-1}$ under (β_i, β_{-i}).

Assume that bidder i's valuation is v_H. If he announces δv_H, his expected payoff is

$$(\delta v_H - \delta v_H + \varepsilon)\text{proba}(\{b_{-i} : b_j \in \{\delta V, \emptyset\}\, \forall j \neq i\}) + \sum_{b_{-i}: b_j = \delta v_H \text{ for some } j \neq i} \text{proba}(\{b_{-i}\}) \frac{\delta v_H - \delta v_H + \varepsilon}{1 + \#H(b_{-i})} > 0.$$

Alternatively, if he announces δV or $\emptyset$, his expected payoff is 0. Hence, regardless of the bidding strategies of his opponents, player i has a unique best response: to announce δv_H.

Assume that bidder i's valuation is v_L (hence has a continuation payoff of V). If he announces δV or $\emptyset$, his expected payoff of announcing. Alternatively, if he announces δv_H, his expected payoff is:

$$(\delta V - \delta v_H + \varepsilon)\text{proba}(\{b_{-i} : b_j \in \{\delta V, \emptyset\}\, \forall j \neq i\}) + \sum_{b_{-i}: b_j = \delta v_H \text{ for some } j \neq i} \text{proba}(\{b_{-i}\}) \frac{\delta V - \delta v_H + \varepsilon}{1 + \#H(b_{-i})} < 0.$$

Thus, in all equilibria, the seller's expected payoff is $(\delta v_H - \varepsilon)(1-(1-\mu)^n) + v_L(1-\mu)^n$.

We next modify the inclusive auction mechanism as follows: If at least two bidders announce δv_H, the successful bidder pays $(\delta v_H - \varepsilon)/\#H(b)$. If exactly one bidder announces δv_H, that bidder pays $\tilde{p}(b_{-i})$, where $\delta v_H - \tilde{p}(b_{-i}) = \frac{\delta v_H - \delta V + 2\varepsilon}{1 + \#L(b_{-i})} > 0$ if $\#L(b_{-i}) \geq 1$ and $\delta v_H - \tilde{p}(b_{-i}) = \delta v_H - \delta V + \varepsilon > 0$ if $\#L(b_{-i}) = 0$. Finally, if all bidders announce either δV or $\emptyset$, a bidder announcing δV pays $(\delta V - \varepsilon)/\#L(b)$ upon acquiring the object.

Assume that bidder i's valuation is v_H. If he reports δv_H, his expected payoff is:

$$\sum_{b_{-i}:b_j=\delta v_H \text{ for some } j\neq i} \text{proba}(\{b_{-i}\})\frac{\delta v_H - \delta v_H + \varepsilon}{1+\#H(b_{-i})} + \sum_{b_{-i}:b_j\in\{\delta V,\emptyset\} \text{ for all } j\neq i} \text{proba}(\{b_{-i}\})(\delta v_H - \tilde{p}(b_{-i})) > 0.$$

Note that it is strictly positive regardless of what others do. Hence, announcing $\emptyset$ is strictly dominated for a bidder with valuation v_H. It follows that there are no equilibria (β_i, β_{-i}), where $\beta_j(v_H) = \emptyset$ for all j, hence the probability that all bidders other than bidder i announces $\emptyset$ is less than one.

Alternatively, if bidder i announces δV, his expected payoff is:

$$\sum_{b_{-i}:b_j\in\{\delta V,\emptyset\} \text{ for all } j\neq i} \text{proba}(\{b_{-i}\})\frac{\delta v_H - \delta V + \varepsilon}{1+\#L(b_{-i})},$$

which is strictly lower than the payoff from truth-telling.

Assume that bidder i's valuation is v_L. If bidder i reports δV, his expected payoff is:

$$\sum_{b_{-i}:b_j\in\{\delta V,\emptyset\} \text{ for all } j\neq i} \text{proba}(\{b_{-i}\})\frac{\delta V - \delta V + \varepsilon}{1+\#L(b_{-i})}.$$

Alternatively, if bidder i reports v_H, his expected payoff is:

$$\sum_{b_{-i}:b_j=\delta v_H \text{ for some } j\neq i} \text{proba}(\{b_{-i}\})\frac{\delta V - \delta v_H + \varepsilon}{1+\#H(b_{-i})} + \sum_{b_{-i}:b_j\in\{\delta V,\emptyset\} \text{ for all } j\neq i} \text{proba}(\{b_{-i}\})(\delta V - \tilde{p}(b_{-i})),$$

where

$$\delta V - \tilde{p}(b_{-i}) = \frac{(\delta V - \delta v_H)\#L(b_{-i}) + 2\varepsilon}{1+\#L(b_{-i})} \leq \frac{\delta V - \delta v_H + 2\varepsilon}{1+\#L(b_{-i})} < 0,$$

if $\#L(b_{-i}) \geq 1$. Thus, reporting v_H gives a strictly negative payoff unless $\text{proba}(\{b_{-i} : b_j = \emptyset\, \forall j \neq i\}) = 1$, which we have ruled out.

Finally, there are no equilibria (β_i, β_{-i}) where $\beta_j(v_L) = \beta_j(v_H) = v_H$ for all $j \neq i$ since any bidder $j \neq i$ with valuation v_L can deviate to $\emptyset$. Therefore,

$$\sum_{b_{-i}:b_j\in\{\delta V,\emptyset\} \text{ for all } j\neq i} \text{proba}(\{b_{-i}\}) > 0.$$

Therefore, in all equilibria, bidders report truthfully their types, which induces the seller's expected payoff:

$$\delta v_H(1-(1-\mu)^{n-1})+\delta V(1-\mu)^{n-1}-\varepsilon(1-(n-2)(1-\mu)^{n-1}\mu) \geq \delta v_H(1-(1-\mu)^{n-1})+\delta V(1-\mu)^{n-1}-\varepsilon,$$

if $n \geq 2$. The expected payoff is $\delta V - \varepsilon$ when $n = 1$. This completes the proof.

6.2 Proposition 3

We first start with a useful lemma.

Lemma 2 *If $b > a \geq 0$ and $d > c \geq 0$, then $(a-d)(b-c) < (a-c)(b-d)$.*

Proof Expanding the inequality we get $ab - ac - db + dc < ab - ad - cb + cd$, or $a(d-c) < b(d-c)$, which is true under the stated hypotheses. □

We now prove the following lemma from which Proposition 3 follows immediately:

Lemma 3 *Consider two networks π and π' such that $L(\pi') > L(\pi)$. We have the following:*

(0) $T(\pi', \delta) < T(\pi, \delta)$.

(1) $V^e(\pi', \delta) < V^e(\pi, \delta)$.

(2) $W^e(\pi', \delta) < W^e(\pi, \delta)$.

(3a) $U^e(\pi', \delta) > U^e(\pi, \delta)$ *if* $T(\pi', \delta) \geq 0$.

(3b) $U^e(\pi', \delta) < U^e(\pi, \delta)$ *if* $T(\pi', \delta) < 0 \leq T(\pi, \delta)$.

(3c) $U^e(\pi', \delta) = U^e(\pi, \delta)$ *if* $T(\pi', \delta) < 0$ *and* $T(\pi, \delta) < 0$.

Proof Part (0) follows from Lemma 2 by letting $a = \delta - \mu$, $b = 1 - \mu$, $c = \sum_n \pi_n (1-\mu)^n = L(\pi)$ and $d = \sum_n \pi'_n (1-\mu)^n = L(\pi')$.

To prove (1), recall the definition of $V \mapsto \mathbb{E}_\pi[R(n, V)]$:

$$\mathbb{E}_\pi[R(n,V)] = \begin{cases} \delta v_H[1 - \sum_n \pi_n (1-\mu)^n] + v_L \sum_n \pi_n (1-\mu)^n & \text{if } V < V^* \\ \delta v_H \frac{1-\mu-\sum_n \pi_n(1-\mu)^n}{1-\mu} + \delta V \frac{\sum_n \pi_n (1-\mu)^n}{1-\mu} & \text{if } V \geq V^*. \end{cases}$$

Note that the definition of V^* is independent of π. Moreover, since $L(\pi') > L(\pi)$, $\mathbb{E}_\pi[R(n,V)] > \mathbb{E}_{\pi'}[R(n,V)]$ for all V (recall that $\delta v_H > v_L$ and $\delta v_H > \delta V$). Moreover, the map $V \mapsto \mathbb{E}_\pi[R(n,V)] - V$ is strictly decreasing. Therefore, since $0 = \mathbb{E}_\pi[R(n, V^e(\pi,\delta))] - V^e(\pi,\delta) > \mathbb{E}_{\pi'}[R(n, V^e(\pi,\delta))] - V^e(\pi,\delta)$, (1) must hold.

To prove (2), assume first that $T(\pi',\delta) \geq 0$, hence $T(\delta,\pi) \geq 0$ by (0). The result then follows directly from the definition of W^e. Assume now that $T(\pi',\delta) < 0 \leq T(\pi,\delta)$. We have that $V^e(\pi',\delta) = W^e(\pi',\delta) < V^e(\pi,\delta) \leq V^e(\pi,\delta) + U^e(\pi,\delta) = W^e(\pi,\delta)$, where the first inequality follows from (1) and the second from $U^e(\pi,\delta) \geq 0$. Finally, if $T(\pi',\delta) < T(\pi,\delta) < 0$, the result follows immediately since $U^e(\pi',\delta) = U^e(\pi,\delta) = 0$.

Finally, (3a)-(3c) follow immediately from the definition of U^e and the observation that the derivative of U^e with respect to $L(\pi)$ is:

$$(\delta - 1)\Big[\frac{1}{(1-\delta L(\pi))^2} - \frac{1-\mu}{(1-\mu-\delta L(\pi))^2}\Big] > 0,$$

when $T(\pi,\delta) > 0$. □

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