Buyer Power and Exclusive Dealing in Bidding Markets*

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Abstract

In bidding markets, suppliers with uncertain costs compete to sell to a single buyer. Let supplier surplus be the total rent obtained by suppliers. If procurement is efficient, a single seller maximizes supplier surplus, although a trade-off exists between cost efficiency and less competition. Instead, multiple suppliers maximize supplier surplus if the buyer runs an optimal auction (i.e., has buyer power) and a condition on the cost distribution holds. We use this result to study the incentives of an upstream firm that licenses suppliers. We show incentives to deal exclusively with a single supplier are smaller with buyer power.

KEYWORDS: Bidding markets, Buyer-Power, Mechanism Design, Input Foreclosure, Exclusive Dealing, Vertical Mergers

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1 Introduction

An upstream firm that controls an input that is essential to the operation of a number of downstream firms selling an homogeneous good may benefit from committing to serve exclusively only one of those downstream firms (e.g. see Hart and Tirole 1990). Such *input foreclosure* allows extraction of monopolistic rents in the downstream market at the detriment of consumers, which may not be possible otherwise. This so-called 'foreclosure doctrine' has informed antitrust scrutiny of many vertical mergers both in Europe and in the US.

There are cases, however, where excluding all but one firm may not be optimal and foreclosure concerns are less serious. For instance, downstream firms may sell differentiated or complement products, be capacity constrained or face decreasing return to scale.[1] In these examples, an upstream firm who negotiates an exclusive deal with some of the downstream firms and extracts their profit encounters a trade-off between suppressing competition and incurring an efficiency loss (e.g., lost sales or costlier production).

We bring an analogous trade-off to surface in the context of *bidding markets* with technologically heterogeneous firms.[2] In our model, a number of suppliers compete to sell an identical product to a single buyer with unit-demand in a procurement auction. Suppliers are ex-ante identical, but have an uncertain cost of production, which for each is the realization of an independent random variable. In terms of the *joint* payoff of all suppliers, having more bidders generates a trade-off between reduced competition and an increase in the possibility of supplying at lower cost. Our central result is that this trade-off is resolved in different ways, depending on whether the buyer has *buyer power* or not, i.e., purchases using an optimally designed procurement mechanism, or using an efficient auction.[3]

In particular, under the standard log-concavity assumption on the distribution of cost, we show that if the buyer runs a second-price procurement auction with no reserve price, which results in an *efficient* allocation, then the joint *supplier surplus* is maximized by having a single bidder, i.e., a monopoly. In turn, if the buyer has buyer power and designs an *optimal* procurement auction, that is a second-price auction with optimally chosen reserve price, then having more than one bidder can maximize joint supplier surplus. We exhibit a simple sufficient condition for this to be the case: the median of the distribution of cost is above the take-it-or-leave-it price that the buyer optimally sets to a single supplier. This condition is satisfied, for instance, by the uniform distribution in the unit interval.

[1] In Condorelli and Padilla (2023) multiple downstream sellers may maximize total profit even if they sell an homogeneous good. If buyers need to make seller-specific investments before purchasing, multiple sellers serve as a commitment that prices will remain low. In turn, this raises the volume of buyers that invest.

[2] Patterson and Shapiro (2001) write "the [European] Commission described a true bidding market as one where tenders take place infrequently, while the value of each individual contract is usually very significant. Contracts are typically awarded to a single successful bidder (so-called winner-takes-all principle)". Klemperer (2007) agrees that this is a common interpretation and adds that "The kind of example often offered as a prototypical bidding market is a large, indivisible, defence contract for a major weapons system".

[3] This modeling of buyer power follows Bulow and Klemperer (1996) and Loertscher and Marx (2019).

The intuition is as follows. Additional competition has two countervailing effects for suppliers. First, it reduces the expected cost draw of the winner. Second, it lowers the expected price paid by the buyer. In an efficient auction, if extreme cost realizations are rare (i.e., log-concavity holds), then the trade-off always resolves with suppliers desiring less competition. Instead, with buyer power the price-lowering effect of an extra bidder is mitigated by the reserve price. Hence, multiple bidders may maximize joint supplier surplus. More precisely, in a second-price auction, an extra supplier with cost between the lowest and second lowest among the current bidders reduces the price without raising efficiency. However, price does not decrease when a reserve price is above the cost of such a supplier.

In the second part of the paper, we consider the implications of the above result for the equilibrium market structure of the supply industry with and without buyer power. To this end, we assume there's a fixed cost that a supplier must pay to enter the market before their cost is realized. First, we analyze the free-entry equilibrium market structure, following Mankiw and Whinston (1986). We observe that, with buyer power, there are less firms active in the free-entry equilibrium than without buyer power. Buyer power reduces the payoff of each individual supplier compared to the no buyer power case, no matter what the number of suppliers is. Hence, with free-entry, buyer power reduces the number of firms that can be active in equilibrium, compared to the case where the procurement auction is efficient.

While not surprising, the free-entry result has the following implication. In light of the well-known Bulow and Klemperer (1996)'s theorem translated to the procurement setting: the buyer is better off running an efficient auction with an extra bidder than running an optimal procurement auction without such bidder. Hence, because of the effect on equilibrium entry, *buyer power may harm the buyer*, unless the buyer is able to commit to an auction format *before* suppliers incur fixed costs.

Next, we consider the case in which the market structure of suppliers is, as in in our opening paragraph, controlled by a monopolistic *upstream* firm that sells them an essential input, which, without loss, we assume it produces at zero marginal cost. Initially, we look at a scenario whereby the upstream firm can impose a fixed payment to suppliers who want to participate in the buyer's auction. This is plausible when, for instance, the upstream firm licenses a technology. Formally, we envisage the upstream firm making simultaneous take-it-or-leave-it offers to all suppliers.

We first show that, if offers made by the upstream firm are *publicly observable*, then the unique equilibrium outcome will enforce the supplier-optimal market structure. That is, the resulting structure will maximize supplier surplus net of fixed costs. The argument is as follows. Since the upstream firm has full bargaining power over suppliers, it will make all of them indifferent between accepting and rejecting their offers. Hence, it will extract in full the surplus of all active suppliers. As a result, the upstream firm will prefer to license the optimal number of suppliers. It follows from our result on supplier optimal market structure that, without buyer power, the upstream firm will want license a single firm and exclude competing firms, while this may not be the case with buyer power.

Then, we look at the case where offers made by the upstream firm are *secret*. That is, each supplier only sees the contract that has been offered to it. This raises additional conceptual issues, due to the asymmetric information that secrecy produces. In short, the equilibrium where supplier surplus is maximized and fully extracted by the upstream firm remains an equilibrium with secret offers, if one is willing to cherry-pick beliefs and assume that an out of equilibrium offer makes the receiver believe that all other potential suppliers have received the same offer. However, suppose beliefs are *passive*, which has been often regarded as a more plausible assumption (see Hart and Tirole (1990) and O'Brien and Shaffer (1992)).[4] Then, in any equilibrium the upstream firm licenses the *free-entry* number of downstream firms. This is more than the optimal number in both buyer and no buyer power cases. Analogously to what we observe in the literature on vertical restraints, with secret contracts the upstream firm loses bargaining power and competition is excessive compared to what maximizes industry profit.

The above result has the following implication for vertical merger policy. In bidding markets, a merger between an upstream firm and one of the downstream firms is less likely to result in inefficient downstream foreclosure when there is buyer power. In fact, suppose we are in the case where a vertical merger might actually result in a change of market structure because, due to the lack of commitment (i.e., as a result of secret contracting), the equilibrium number of firms is larger than the one desired by the upstream firm. On the one hand, without buyer power exclusion of all downstream competitors is always profitable following the merger. On the other hand, with buyer power the merger might not result in any downstream foreclosure. In fact, keeping a larger than ideal number of competing suppliers in the market may be more profitable for the merged entity than complete exclusion.

In the preceding analysis, an upstream firm decides whether to license each supplier at a fixed price. In practice, more complex contracts may be written between the upstream firm and suppliers. In the final part of the paper, we investigate how such contracts can be used by the upstream firm to increase and extract the surplus obtained by suppliers in the buyer's auction, with and without buyer power. Our main result is that an upstream firm able to commit to public offers that require a payment only by the winning supplier will always strictly benefit from contracting with more suppliers. In fact, in the limit as the number of downstream suppliers becomes large, the upstream firm extracts the entire surplus by setting a payment for the winning firm which is equal to the (known) value for the buyer minus the lowest possible cost. With buyer power, this will force the upstream firm to increase the reserve price. Yet, due to the winning fee, the contract will be profitable only for a supplier with a very low cost. However, because the number of suppliers is very large, the upstream firm can count on a very low cost supplier participating to the auction. Since the upstream firm can extract the first-best surplus, there's no motive for exclusion in this case.

[4]That is, following reception of out-of-equilibrium offer, a supplier's beliefs about the offers made to other suppliers remains those such a supplier has along the candidate equilibrium path.

This observation extends to bidding markets the classic conclusion that vertical integration and exclusive are not needed to enforce a downstream monopoly if the upstream firm can commit to its offers and has access to sufficiently flexible pricing. In that case, the upstream firm can normally enforce the desired monopolistic price level and extract the surplus by possibly trading with all downstream firms but committing to optimally designed two-part tariffs.

This paper speaks to two branches of the economics literature. From an applied perspective, our results in Section 5 and 6 contribute to the analysis of buyer power, exclusive dealing and vertical foreclosure.[5] Methodologically, we borrow the mechanism design framework to analyze buyer power from Loertscher and Marx (2019). They study how buyer power affects horizontal mergers of suppliers in bidding markets. Their main result is that, buyer power or not, a merger without cost synergies harms the buyer. In follow up work, Loertscher and Marx (2022), they also consider vertical mergers between buyers and suppliers. They emphasize that while a merger between a single buyer and a single seller is always welfare enhancing, a merger between a supplier and one of many buyers may exacerbate asymmetries of information, thus reducing welfare. By introducing an upstream licensor of suppliers in the picture, we are able to discuss vertical mergers within different levels of the supply chain and input foreclosure. We believe our analysis complements theirs.

From a more theoretical perspective, we see our Propositions 2 and 3 as contributions to the mechanism design literature. The existing research on auction theory is vast and spans across economics and computer science (e.g. see Krishna (2010) and Milgrom (2004)). However, there is no prior work that we are currently aware of that identifies the number of buyers that maximize total buying surplus in an efficient and an optimal auction in the canonical independent private-value environment.[6] Beyond the study of buyer power, our results may also help shed light on other applied issues, such as collusion in auctions. In fact, Proposition 1 indicates that there is always a price that the one firm is willing to pay and the other is willing to accept ex-ante for one of them to remain out of an efficient auction. Proposition 2 suggests that this might not be the case in an optimal auction.

The plan for the rest of the paper is the following. First we present the environment and the mechanism design framework. In Section 4 we state our main results on the supplier-optimal number of suppliers. In Section 5 we study equilibrium market structure. Section 6, which concludes the paper, discusses more complex contracts between the upstream firm and suppliers.

[5] The literature on these topic is vast. A very incomplete list of recent contributions includes Rey and Whinston (2013), Calzolari and Denicolò (2015), Hansen and Motta (2019), Calzolari, Denicolò and Zanchettin (2020), Chambolle and Molina (2023). For an excellent textbook treatment see Fumagalli, Motta and Calcagno (2018).

[6] As it is well known, the procurement problem is isomorphic to the selling problem, so results can be naturally translated between the two frameworks.

2 Environment

A buyer, denoted B, is looking to procure a single indivisible product that it values at v. Let $n \geq 1$ be the number of potential downstream suppliers of the product. Later on, we will make n endogenous. Each supplier $i \in \{1, \ldots, n\}$ has opportunity cost c_i for providing the product, which is independently drawn from the same distribution and is private information of that supplier. We assume that the CDF of cost is G, has full support in the interval $[\underline{c}, \overline{c}]$ and admits bounded density g. Without loss of generality, we assume $v = \overline{c} > \underline{c} \geq 0$. If a purchase takes place between i and B at price p, then the payoff of B is $v - p$ and the payoff of supplier i is $p - c_i$. The payoff of agents who do not trade is zero.

Our analysis compares two alternative scenarios. In one, we assume B has *no buyer power* and runs a second-price procurement auction *without* reserve price. The second-price auction has the property that it allocates the contract for supplying the product efficiently, that is always to the lowest cost supplier. In another, we assume B has *buyer power* and sets a second-price procurement auction *with* optimally chosen reserve price. With symmetric bidders, as long as the reserve price is optimally chosen, the auction maximizes the surplus of the buyer among all possible auctions. This captures a situation in which the buyer has all bargaining power. We formally describe the two auction mechanisms in the next section.

3 Mechanism design preliminaries

In this section, we take n as given and compute expected payoffs in the auction, for the buyer and active suppliers, in both the no buyer power and buyer power scenarios. To do so, we adopt a classic mechanism design approach, in the tradition of Myerson (1981). The section contains no new results and a reader familiar with mechanism design can skim through it.

Formally, a *mechanism* chosen by the buyer is a normal-form Bayesian game that is played by the suppliers, who have private information about their cost. Each supplier submits a non-negative bid and the mechanism determines, as function of the profile of bids, who is awarded the contract, if any, and what price the winner receives from the buyer in exchange.[7] Payoffs are determined based on the outcome of the mechanism, given the preferences outlined in the previous section.

A mechanism that is acceptable for the buyer and suppliers must be (interim) *individually rational.* We formalize this notion by requiring that (i) the payment from the buyer to the winning supplier never exceeds v and (ii) that for each supplier there is at least one bid that guarantees either losing the contract or, otherwise, being paid $\overline{c} = v$ for it.[8] As it will

[7] A mechanism may allow for an arbitrary space of bids and require payments also to and from non-winning bidders. Ruling these possibilities out is without loss of generality for us and simplifies the exposition.

[8] The traditional definition of individual rationality requires that in the *equilibrium* of the mechanism that is being considered, B expects to pay at most v for the contract and all bidders expect to at least break-even.

become clear later, both the second-price procurement auction with reserve price and the one without are individually rational mechanisms.

If a mechanism admits an equilibrium, then we can define the *allocation rule* of the mechanism as mapping each profile of costs (i.e., profiles of Bayesian types of players) into a lottery over the allocation of the object that is implied by equilibrium play given the profile of costs. Hence, an allocation rule of an equilibrium of a mechanism is a function

$$\boldsymbol{q} : [\underline{c}, \overline{c}]^n \to \Delta[0,1]^{n+1} \tag{1}$$

where $q_i(c_1, \ldots, c_n)$ stands for the probability that the contract is awarded to $i = 1, \ldots, n$ given that the profile of costs is $(c_1, \ldots, c_n)$ and $q_0(c_1, \ldots, c_n)$ is the probability that the buyer does not buy at all.

A central result in mechanism design theory, the Payoff Equivalence Theorem (see Myerson (1981) and Milgrom (2004), Theorem 3.3., for a textbook treatment) implies that, in our environment, the payoff of the auctioneer and that of suppliers in any equilibrium of an individually rational mechanism can be uniquely identified via its interim allocation rule.[9]

Appealing to the Payoff Equivalence Theorem, the expected payoffs of buyers and suppliers can be written as a function of the induced allocation rule alone. To this end, we introduce an additional element of the mechanism design approach to the procurement problem, the *virtual cost*. The virtual cost is a function $\Gamma : [\underline{c}, \overline{c}] \to [\underline{c}, \infty)$ that is formally defined as

$$\Gamma(c) = c + \frac{G(c)}{g(c)}. \tag{2}$$

Since g is finite, we observe that $\Gamma(\underline{c}) = \underline{c}$ because $G(\underline{c}) = 0$ and $\Gamma(\overline{c}) > \overline{c} = v$. An interpretation of the virtual cost is that it represents the (marginal) price the buyer must pay to procure the product from a supplier with a certain cost by means of a mechanism, considering the additional mark-up it must provide to such buyer type (i.e., the the second term in (2)).Using known mechanism design techniques we obtain the following result.

Proposition 1 *Suppose B uses an individually rational mechanism and let $\boldsymbol{q}$ be a function as in (1) that represents the (relevant) equilibrium allocation rule of such mechanism.*[10] *Then, the equilibrium buyer surplus, $BS(\boldsymbol{q})$, is given by*

$$BS(\boldsymbol{q}) = \mathbb{E}_{\boldsymbol{c}} \left[\sum_i q_i(\boldsymbol{c})(v - \Gamma(c_i)) \right],$$

Assumptions (i) and (ii) restrict the set of feasible mechanisms without loss of generality, guaranteeing that this is the case.

[9]In addition to having the same allocation rule, the Payoff Equivalence theorem also requires that the two equilibria give the worst-off type the same utility. As a result of assumptions (i) and (ii) any individually rational mechanism will give suppliers with cost $\overline{c}$ a payoff of zero.

[10]A mechanism may admit multiple equilibria. The proposition applies to all such equilibria.

and the sum of ex-ante expected payoffs of all suppliers, called supplier surplus and denoted with $SS(\boldsymbol{q})$, is equal to

$$SS(\boldsymbol{q}) = \mathbb{E}_{\boldsymbol{c}}\left[\sum_i q_i(\boldsymbol{c})(\Gamma(c_i) - c_i)\right].$$

Proof The proof follows from standard arguments and is omitted. See Loertscher and Marx (2019) for an explicit derivation. □

Two remarks are in order. First, because payments are welfare-neutral, the total welfare generated ex-ante by an individually rational mechanism with interim allocation rule $\boldsymbol{q}$, denoted $W(\boldsymbol{q})$, is

$$W(\boldsymbol{q}) = BS(\boldsymbol{q}) + SS(\boldsymbol{q}) = \mathbb{E}_{\boldsymbol{c}}\left[\sum_i q_i(\boldsymbol{c})(v - c_i)\right].$$

Second, as long as $\boldsymbol{q}$ represents an equilibrium of some mechanism, then $\mathbb{E}_{c_{-i}}[q_i(c_i, \boldsymbol{c}_{-i})]$, called the *interim allocation rule of* i, must be non-increasing for all $i = 1, \ldots, n$. That is, lower cost suppliers should expect to be awarded the contract more often than if they had higher cost (Myerson (1981)). This observation will be relevant to understand the construction of the optimal mechanism for the buyer.

With Proposition 1 in hand, we now derive payoffs in both the *no buyer power* and in the *buyer power* scenarios.

No-Buyer-Power: Efficient Auction. In an *efficient procurement mechanism* (i.e., the second-price auction without reserve price), all suppliers simultaneously and independently bid a non-negative amount for the contract and the supplier who submits the lowest bid is awarded the contract at a price equal to the second-lowest bid or v, whichever is smaller. Ties are resolved randomly. It is easy to see that this mechanism is individually rational, as the maximum payment the buyer may make is v and bidding above v always guarantees a non-negative payoff even to a supplier with cost $\bar{c}$.

As it is well-known, in the equilibrium in undominated-strategies of this mechanism, on which we henceforth focus, each supplier bids its own cost.[11] Therefore, the lowest cost supplier always wins the auction. Because $\bar{c} = v$ (i.e., there are almost always gains from trade), the auction results in an efficient outcome. Let $\#_{min}(\boldsymbol{c}) = \#\{i : c_i = \min\{c_1, \ldots, c_n\}\}$ indicate the number of suppliers who have the minimum cost; (this notation is regrettably required even though ties have zero probability). We conclude that the allocation rule for the efficient second-price auction, denoted $\boldsymbol{q^e}$, is defined by

[11] Such a second-price auction also admits "collusive" equilibria. However, the competitive equilibrium we consider is prominent for a variety of reasons, last but not least the fact that it is the only one under which all bidders play an undominated strategy.

$$q_i^e(\boldsymbol{c}) = \begin{cases} \frac{1}{\#_{min}(\boldsymbol{c})} & \textit{if } c_i = \min\{c_1, \dots, c_n\} \\ 0 & \textit{otherwise.} \end{cases}, \textit{ for } i = 1, \dots, n. \quad (3)$$

We can now use $\boldsymbol{q^e}$ to obtain the supplier surplus in the second-price auction without reserve price. Letting $c_{(1:n)}$ be the random variable indicating the minimum out of n independent draws from G and letting $G_{(1:n)}$ indicate its CDF, we can state the following as a corollary to Proposition 1.

Corollary 1 *In a second-price procurement auction the supplier surplus, denoted SS_n^e, is*

$$SS_n^e = \mathbb{E}_{\boldsymbol{c_{(1:n)}}}\left[\frac{G(c_{(1:n)})}{g(c_{(1:n)})}\right] = \int_{\underline{c}}^{\overline{c}} \frac{G(x)}{g(x)} dG_{(1:n)}(x).$$

Proof

$$SS(\boldsymbol{q}) = \mathbb{E}_{\boldsymbol{c}}\left[\sum_i q_i^e(\boldsymbol{c}) \frac{G(c_i)}{g(c_i)}\right] = \mathbb{E}_{\boldsymbol{c}}\left[\frac{G(\min\{c_1, \dots, c_n\})}{g(\min\{c_1, \dots, c_n\})}\right] = \mathbb{E}_{\boldsymbol{c_{(1:n)}}}\left[\frac{G(c_{(1:n)})}{g(c_{(1:n)})}\right].$$

□

Buyer Power: Optimal Procurement Auction We now consider the second-price procurement auction with optimally chosen reserve price below v. In such auction, bidders submit bids and the supplier who submitted the lowest bid among those that are below the *reserve price* is awarded the contract at a price equal to the second-lowest bid or the *reserve price*, whichever is smaller. Again, ties are resolved randomly. Because the reserve price is below v and because by bidding above the reserve price each supplier can make sure not to sell to the buyer, this mechanism is individually rational.

We first show that, as long as the reserve price is optimally chosen, this auction maximizes the buyer's surplus. We will then write down the allocation rule for the auction with the optimally chosen reserve price and obtain the supplier-surplus under such an auction.

While it is possible, given the stated assumptions, to characterize an optimal auction for any arbitrary distribution, we focus now on what in the mechanism design literature is known as *regular case.*

Assumption 1 *The reverse hazard rate of G, that is g/G, is monotone decreasing.*

A mechanism design problem with a distribution that satisfies this assumption is called "regular" because of tractability (e.g., it guarantees the buyer's optimal mechanism takes the simple form identified below) and because many distribution employed in applied work satisfy this monotone hazard-rate property, including the uniform and the Gaussian.

Assumption 1 implies that G/g is monotone increasing and, as a consequence, that also Γ is monotone increasing. This has an important consequence. Inspection of $BS(\boldsymbol{q})$ from Proposition 1 indicates that the *unconstrained* allocation rule that maximizes supplier-surplus awards the contract, for any given $(c_1, \ldots, c_n)$, to an agent with the minimum cost, let's call it i_m, as long as $v - \Gamma(c_{i_m}) \geq 0$, that is, $c_{i_m} \leq \hat{c} = \Gamma^{-1}(v)$.[12] Otherwise, the contract should not be awarded and it is optimal to have $q_0(\boldsymbol{c}) = 1$.[13]

Because it is well known that, under the stated assumptions, the second-price procurement auction with reserve price $\hat{c}$, has an equilibrium with the allocation rule described above, we conclude that such an auction maximizes buyer surplus among the set of all individually rational procurement mechanisms. Remarkably, because $v < \Gamma(\bar{c})$ but $c_{i_m} \leq \bar{c}$, optimal procurement results sometimes in an inefficiency, in contrast to what happens under a second price procurement auction without reserve price.

Letting $\#_{min \leq \hat{c}}(\boldsymbol{c}) = \#\{i : c_i = \min\{c_1, \ldots, c_n, \hat{c}\}\}$, this optimal allocation rule, let's denote it $\boldsymbol{q}^o$, is more formally defined as

$$q_i^o(\boldsymbol{c}) = \begin{cases} \frac{1}{\#_{min \leq \hat{c}}(\boldsymbol{c})} & \textit{if } c_i = \min\{c_1, \ldots, c_n, \hat{c}\} \\ 0 & \textit{otherwise.} \end{cases}, \quad \textit{for } i = 1, \ldots, n.$$

We can now apply Proposition 1 to $\boldsymbol{q}^o$ to obtain the supplier surplus in the optimal procurement auction.

Corollary 2 *Under Assumption 1, in a second-price procurement auction with reserve price $\hat{c}$ the supplier surplus, SS_n^o, is equal to*

$$SS_n^o = \Pr\{c_{(1:n)} \leq \hat{c}\} \mathbb{E}_{\boldsymbol{c}_{(1:n)}} \left[\frac{G(c_{(1:n)})}{g(c_{(1:n)})} \mid c_{(1:n)} \leq \hat{c} \right] = \int_{\underline{c}}^{\hat{c}} \frac{G(x)}{g(x)} dG_{(1:n)}(x).$$

Proof Standard mechanism design arguments (e.g., see Lemma 2 in Myerson (1981)) imply that the expected surplus of a supplier with cost c is

$$\int_c^{\bar{c}} Q_i^o(x) dx$$

where $Q_i^o(x) = \mathbb{E}_{c_{-i}}[q_i^o(x, \boldsymbol{c}_{-i})]$ is the probability that the contract is awarded to i when it has cost c. Because with symmetric suppliers the probability that i with cost x wins the contract

[12] $\hat{c}$ is well defined because the inverse Γ^{-1} exists since Γ is strictly monotone. Remarkably, $\hat{c}$ is the price that solves the optimization problem of the buyer when it makes a take-it-or-leave-it offer to a single supplier. In fact, B solves $\max_p (v - p)G(p)$, where p is the price it offers to the supplier and the supplier accepts the offer when p is above its realized cost. Assuming $v < \bar{c}$, the optimal price is characterized by the first-order condition $0 = g(p)\left(v - p + \frac{G(p)}{g(p)}\right)$.

[13] This is the unconstrained optimum because it is the result of pointwise maximization of buyer surplus, without regard for the incentive compatibility requirement that the interim allocation be decreasing.

is equal to $\Pr\{x \leq c_{(1:n-1)}\} = (1 - G(x))^{n-1}$ if its cost is below $\hat{c}$ and is zero otherwise, i's expected payoff is

$$\int_c^{\hat{c}} (1 - G(x))^{n-1} dx.$$

Then, the supplier surplus is obtained by averaging over the possible cost values and summing over the various bidders. Because suppliers are symmetric we have

$$SS_n^o = n \int_{\underline{c}}^{\hat{c}} \int_c^{\hat{c}} (1 - G(x))^{n-1} dG(x).$$

Changing the order of integration and then dividing and multiplying by $g(x)$

$$SS_n^o = n \int_{\underline{c}}^{\hat{c}} G(x)(1 - G(x))^{n-1} dx = \int_{\underline{c}}^{\hat{c}} \frac{G(x)}{g(x)} n g(x)(1 - G(x))^{n-1} dx,$$

which is the desired expression as $ng(x)(1 - G(x))^{n-1}$ is the density function of $c_{(1:n)}$. □

Since $\hat{c} < v = \bar{c}$ by looking at their integral expressions it is immediate to see that $SS_n^o < SS_n^e$ for all $n \geq 1$. Naturally, $BS_n^o > BS_n^e$ while total welfare is maximized when a reserve price is absent, that is (introducing new but obvious superscript notation) $W_n^e > W_n^o$.

4 Supplier Surplus Maximizing Industry Structure

In this section, we present a central result of this paper. We show that, under Assumption 1, if there is no buyer power, then supplier surplus is maximized when only one firm supplies the product. However, when there is buyer power, supplier surplus will be maximized by more than one firm being active if the median of G is above the reserve price. We state these two results in turn, beginning with the no buyer power scenario.

Proposition 2 *If the reverse hazard rate g/G is monotone decreasing (Assumption 1), then the supplier surplus with no buyer power is maximized in the presence of a single supplier. That is, $SS_1^e \geq SS_n^e$ for all $n > 0$.*

Observe that the proof of the Proposition also shows that if, in contrast to Assumption 1, the reverse hazard rate g/G is monotone increasing, then the supplier-surplus SS_n^e increases in the number of suppliers n. However, in practice, the optimal number of suppliers is not infinite because the reverse hazard rate is always going to be decreasing in the neighborhood of the lower bound of the support, unless one is willing to assume $\underline{c} = -\infty$.

Proof $c_{(1:n)} \succ_{fod} c_{(1:n+1)}$ for all $n \geq 1$, that is, the minimum of n independent draws first-order stochastically dominates the minimum of $n+1$ draws. In fact, $G_{(1:n)}(c) = 1 - [1 - G(c)]^n$ and $1 - [1 - G(c)]^n \leq 1 - [1 - G(c)]^{n+1}$. Then, note that the expectation of a non-increasing function of a non-negative real-valued random variable, always decreases with a first-order stochastic shift of the distribution (see Mas-Colell, Whinston and Green (1995), Proposition 6.D.1). We conclude that $SS_n^e = \mathbb{E}_{\boldsymbol{c_{(1:n)}}}\left[\frac{G(c_{(1:n)})}{g(c_{(1:n)})}\right]$ is decreasing in n. $\square$

A partial intuition for the above result would be the following. From the joint perspective of suppliers, having one more supplier generates a trade-off between the decrease in the expected payment from the buyer that arises from more competition *and* the expected efficiency gain from sampling an additional cost draw. When buyers' values are likely to be close to each other, additional competition is hurtful and does not generate much efficiency gain. Hence, the trade-off resolves in the direction of desiring less competition.

Under Assumption 1, supplier surplus is not aligned with buyer surplus nor, somewhat more surprisingly, with welfare. In fact, total welfare always increases in the number of suppliers operating in the market (i.e., assuming any costs that suppliers may have paid to enter the market are sunk). So does buyer surplus.

We consider next the case of buyer power. As it turns out, even if Assumption 1 is satisfied, it is not guaranteed that a single supplier maximizes supplier surplus. The trade-off described above sometimes resolves in the direction of having additional suppliers. We have the following result.

Proposition 3 *If the reverse hazard rate g/G is monotone decreasing (Assumption 1), then the supplier surplus with buyer power is maximized by having more than one supplier if $G(\hat{c}) \leq 1/2$, that is $\hat{c}$ is below the median of the distribution of cost.*

Note that this proposition does not rule out the possibility that the supplier surplus is maximized by more then one firm when the median of the distribution is below the reserve price.

Proof We can rewrite SS_n^o as

$$\Pr\{c_{(1:n)} \leq \hat{c}\}\mathbb{E}_{\boldsymbol{c_{(1:n)}}}\left[\frac{G(c_{(1:n)})}{g(c_{(1:n)})} \mid c_{(1:n)} \leq \hat{c}\right] = \int_{\underline{c}}^{\hat{c}} \frac{G(x)}{g(x)} dG_{(1:n)}(x) = \int_{\underline{c}}^{\hat{c}} G(x)n[1-G(x)]^{n-1}dx$$

We can then ask when a monopoly is worse than having n firms as suppliers. Given that $\hat{c}$ is independent of the number of firms, this is when

$$\int_{\underline{c}}^{\hat{c}} G(x)n[1-G(x)]^{n-1}dx \geq \int_{\underline{c}}^{\hat{c}} G(x)dx$$

hence when $n = 2$ this amounts to asking whether the following is true

$$\int_{\underline{c}}^{\hat{c}} G(x)[1-2G(x)]dx \geq 0.$$

Then note that the condition $G(\hat{c}) \leq 1/2$ implies that $[1 - 2G(x)]$ is positive in $[\underline{c}, \hat{c}]$ as G is increasing. Therefore the inequality is true. The fact that having two downstream firms creates more supplier surplus than one firm shows that a monopolistic supplier is not optimal with buyer power. $\square$

Assumption 1 and the condition of the proposition are satisfied in a number of natural cases. For instance, consider the uniform distribution in $[0, 1]$. We have previously mentioned that G/g is increasing in this case. Therefore, without buyer power, a single supplier maximizes supplier-surplus. However, the uniform distribution also satisfies the condition from the proposition above. That is, $\hat{c} = 1/2$. Therefore a monopolistic supplier does not maximize supplier surplus when there is buyer power if the distribution of cost is uniform.

In contrast to the case with no reserve price, the price-lowering effect of an additional bidder is smaller because the buyer withdraws demand when the price is too high by setting a price ceiling equal to the reserve price. Hence, compared to the no buyer power case, it's now less likely that a cost draw that does not improve efficiency will end up inducing a lower price.

5 Equilibrium market structure

In this section, we derive implications of the previous results for market structure. To begin with, we assume entry requires suppliers to sustain, before their cost is realized, a sunk cost $k > 0$, which is smaller than the monopoly profit of a single supplier under buyer power, i.e., $k < SS_1^o \leq SS_1^e$. We focus on two scenarios, a free entry scenario and one in which the market is controlled by an upstream monopolist, U, which offers an input essential for suppliers to participate in procurement. We conclude the section by developing an example where costs are drawn from the uniform distribution.

Free-entry. Suppose entry in the supplier's market is free. In a classic free-entry equilibrium (e.g., see Mankiw and Whinston (1986)), the profit that each active supplier obtains must be sufficient to cover its fixed cost, but any additional entrant must be faced with a loss. Since firms are symmetric, all active firms expect the same profit from the procurement stage. Therefore, if n^* is the free-entry equilibrium number of firms, then the average net surplus accrued by n^* firms should be positive, while that accrued by $n^* + 1$ firms should be negative.

Observing that both SS_n^e/n and SS_n^o/n are decreasing in n (i.e., an extra bidder never benefits other bidders in an auction) and abstracting from the fact that the equilibrium number must be an integer, we conclude that *without buyer power*, the equilibrium number of firms, denoted n_f^e, solves

$$SS^e_{n_f^e}/n_f^e = \int_{\underline{c}}^{\overline{c}} G(x)[1-G(x)]^{n_f^e-1}dx = k.^{14}$$

In the case of *buyer power*, in which the buyer uses an optimal auction, the equilibrium number, n_f^o solves

$$SS^o_{n_f^o}/n_f^o = \int_{\underline{c}}^{\hat{c}} G(x)[1-G(x)]^{n_f^o-1}dx = k.$$

Wrapping up, we make the following observation.

Proposition 4 *Consider a free-entry equilibrium with entry cost k and denote the equilibrium number of firms with and without buyer power, n_f^o and n_f^e respectively. Then, there is more entry without buyer power than with buyer power, that is $n_f^e \geq n_f^o$.*

Proof This is simply because for every n

$$\int_{\underline{c}}^{\hat{c}} G(x)[1-G(x)]^{n-1}dx \leq \int_{\underline{c}}^{\overline{c}} G(x)[1-G(x)]^{n-1}dx$$

since $\hat{c}$ is independent of n and $\hat{c} < \overline{c}$. □

Buyer power reduces the payoff of each individual supplier compared to the no buyer power case, no matter what the number of suppliers is. Hence, buyer power reduces the number of firms that can be active in equilibrium, even if there's free entry.

Proposition 4 also has the following, somewhat counterintuitive, implication. In light of the Bulow and Klemperer (1996) theorem (translated to the procurement setting), the buyer is better off from running an efficient auction with an extra bidder than it is from an optimal auction without such bidder. That is, $BS^e_{n+1} > BS^o_n$. Hence, because of the effect on equilibrium entry, buyer power may harm the buyer, unless such buyer is able to commit to an auction format *before* suppliers incur fixed costs. In particular if $n_f^e > n_f^o$ a buyer would prefer to commit, before entry decisions are made, to an efficient auction without reserve price in order to guarantee additiona; entry. Of course, such a commitment may not be possible.

To conclude we observe that, clearly, free-entry destroys the rent of suppliers, except possibly for the rent that is accrued because the number of firms must an integer.

Upstream Control. Now suppose a monopolistic upstream firm, denote it with U, controls an input essential for suppliers to operate in the market. Bargaining between U and a large number $\overline{n}$ of potential suppliers takes place ex-ante, that is before suppliers learn

[14] More formally the n^e integer solves $\int_{\underline{c}}^{\overline{c}} G(x)[1-G(x)]^{n^e-2}dx \geq k \geq \int_{\underline{c}}^{\overline{c}} G(x)[1-G(x)]^{n^e-1}dx$.

their cost and before they sustain their fixed cost k. We assume $\overline{n}$ is large, more formally $k > SS^e_{\overline{n}}/\overline{n}$. The upstream firm makes simultaneous take-it-or-leave-it offers to the $\overline{n}$ suppliers. To each supplier i, U offers a price t_i for the right to operate in the supplier-market. Suppliers simultaneously and independently decide whether to accept or reject their offers. If a supplier decides to accept, it pays the fixed price to U, and, sustaining fixed cost k, it can participate in the buyer's auction. If it rejects, then there's no payment to U and such a supplier stays out of the market. We assume the upstream firm has zero marginal cost or, more generally, that the cost sustained is independent of the number of licensed firms.[15]

Following McAfee and Schwartz (1994) and the large literature on vertical contracting, we will consider two cases separately.[16] First, we analyze the case where offers made by U are *public* and observable. This scenario provides U with commitment and, therefore, full bargaining power. Second we consider the case where offers are *secret*, that is each supplier only observes the offer that has been made to it. This, as we shall see, mitigates the bargaining power of U vis-a-vis the suppliers.

The next proposition states that, if offers are observable, the unique equilibrium outcome is such that U will offer to the the number of suppliers that maximize supplier surplus net of fixed costs depending on the scenario. More formally, we have the following.

Proposition 5 *Assume offers by U are publicly observable and the reverse hazard rate g/G is monotone decreasing (Assumption 1). Then, when there is no buyer power, U will ask*

$$t^e = SS^e_1 - k$$

to one supplier, while making unacceptable offers to others. The supplier who receives an offer at t^e accepts it, while others refuse their offers.

If there is buyer power, then U will ask

$$t^o = \frac{SS^o_{n^o_m} - n^o_m k}{n^o_m} \text{ where } n^o_m = \arg\max_{n \in \mathcal{N}^+} \{SS^o_n - nk\}$$

to n^o_m suppliers, while making unacceptable offers to others. Suppliers who receive an offer at t^o accept it, while others refuse their offers.

This result is straightforward and therefore stated without proof. Since U has full bargaining power over suppliers, it will make all of them indifferent between accepting and rejecting their offers. Hence, it will extract in full the surplus of all active suppliers. As a result, U will prefer to license the optimal number of suppliers. It follows from Proposition 2 that, without buyer power, the upstream firm will license a single firm, while this may not be the case with buyer power.

[15] Assuming otherwise would complicate the analysis without bearing new insights.

[16] See also Hart and Tirole (1990), O'Brien and Shaffer (1992) and Rey and Verge (2004).

We now consider the case of secret contracts. Following McAfee and Schwartz (1994), this scenario captures absence of commitment by the upstream firm. Now, when a supplier decides whether to accept the offer or not, it is unsure about which other firms are entering the market and U cannot credibly offer that information. We focus on the plausible case in which who else is competing in the auction of the buyer becomes observable before the auction.[17] We will now take as given payoffs in the auction as a function of the market structure and focus on the ex-ante negotiation between U and suppliers. The equilibrium concept is Perfect-Bayesian equilibrium.

Under secret contracts, multiple equilibria are possible. While suppliers must have correct beliefs about equilibrium offers, after receiving an out-of-equilibrium offer by U a supplier can form arbitrary beliefs about what other offers U has made. Hence, equilibria can be sustained by attributing to suppliers ad-hoc, potentially implausible, beliefs that lead to rejection of out-of-equilibrium offers that would benefit U but upset equilibrium. In fact, the first observation we make is precisely that there is always an equilibrium where secret offers are as in Proposition 5.

Proposition 6 *Suppose that following an out-of-equilibrium offer to i, then i believes that all potential suppliers are receiving the same offer. The commitment offers from Proposition 5 are an equilibrium supported by such beliefs.*

Proof Suppose the upstream firm deviates offering some different contract to firm i. Then i believes that all $\overline{n}$ suppliers have received the same offer. However, $k > SS^e_{\overline{n}}/\overline{n}$ and therefore i is better off refusing the offer, as long as U asks a positive amount of money. Of course a deviation where U offers a payment to i is not a profitable deviation for U. □

In short, the equilibrium where supplier surplus is maximized and fully extracted by U remains an equilibrium with secret offers if one is willing to assume that an out of equilibrium offer makes the receiver believe that all other potential suppliers have received the same offer. Note, however, that any profile of offers that gives a non-negative profit to U is also an equilibrium under the beliefs above. In fact, any potentially profitable deviation by U would be met with a refusal. This suggests that anything-goes beliefs of this sort are somewhat implausible.

In order to perform equilibrium selection, the literature has therefore focused on restricting the set of permissible beliefs. One such common restriction is the assumption of passive beliefs. Under passive beliefs a firm who receives an out-of-equilibrium offer does not update its beliefs about the offers made to other firms, which remain the equilibrium ones. See McAfee and Shwartz (1994) for more on the relevance of passive beliefs. Under this restriction, it becomes easier for an out-of-equilibrium offer to be acceptable. Then, it

[17] Because it restores commitment, U may want to write contracts conditional on who participates to procurement. But such contracts are difficult to enforce and unlikely to be legal in practice.

is easy to see that the profile of offers from Proposition 5 is no longer an equilibrium. The upstream firm can always deviate to licensing an additional firm.[18] Extending this reasoning further, we have the following result.

Proposition 7 *Suppose beliefs are passive. Then, if the assumptions of Proposition 4 hold, in any equilibrium the upstream firm licenses the free-entry number of firms characterized in Proposition 4. This is more than the optimal number in both cases, unless $\overline{n} < n_m^o$.*

Proof By way of contradiction, suppose in equilibrium the number of licensed firms is n and is below the free entry equilibrium level. Then U could deviate by making an additional offer to another firm at price $SS_n^s/n - k$ and such offer would be accepted. The number of licensed firms cannot be above the free-entry equilibrium level or otherwise either U or at least one supplier would make a loss. □

With secret contracts, U loses its bargaining power and entry is excessive. This suggests that absence of commitment is more likely to harm U when there is no buyer power. Interestingly, a merger between U and firm 1, with no buyer power, would restore commitment for U and therefore would be anticompetitive in this model, because it would raise prices to the buyer. As our result shows, this is not necessarily the case with buyer power. There are two forces at play. First, the number of active firms will be lower pre-merger with buyer power. Second, having more than one downstream firm active often maximizes supplier surplus, and therefore also the joint surplus of 1 and U, especially if the latter is able to achieve its commitment outcome.

We conclude this section with a numerical example where we assume that G is a uniform distribution in $[0, 1]$.

Uniform Example We perform the computation under the uniform distribution with $G(x) = x$ for $x \in [0, 1]$. The supplier surplus under the efficient auction is

$$SS_n^e = \int_0^1 G(x)n[1 - G(x)]^{n-1}dx = n\int_0^1 x[1 - x]^{n-1}dx = \frac{1}{1+n},$$

and it is immediate to see that SS_n^e is strictly decreasing. Hence, it is maximized at $n = 1$.

If G is a uniform distribution, then the optimal reserve price is $\hat{c} = 1/2$. The supplier surplus under the optimal auction is

$$SS_n^o = \int_0^{1/2} G(x)n[1 - G(x)]^{n-1}dx = n\int_0^{1/2} x[1 - x]^{n-1}dx = \frac{1 - \frac{1}{2}^n\left(\frac{n}{2} + 1\right)}{n+1},$$

which is maximized by $n_m^o = 3$ among the integer numbers.

[18] For the buyer power case this true is unless $n_m^o = n_f^o$, that is having an additional firm would bring the total number of firms to a level that would not allow firms to cover fixed costs.

Now suppose there is a fixed cost of entry of 1/30, which is about 6.5% of the average cost of the entire contract for suppliers. Then, in a free-entry equilibrium, the number of firms will be equal to 4 with buyer power and 5 without buyer power.

Next assume fixed cost of entry are still equal to 1/30 but the upstream firm controls entry. If U has commitment power, then according to Proposition 5 it will license 2 firms if the there is buyer power, while it will create a monopolist supplier if there is no buyer power. Under secret contracts and passive beliefs, the outcome is the free-entry one.

6 Complex Licensing

In this section, we allow the upstream firm to make public offers that require a payment only by the winning firm. Our main result is that, in this case, an upstream firm will always *strictly* benefit from contracting with more suppliers. Indeed, in the limit as the number of downstream suppliers becomes large, the upstream firm extracts the entire first-best surplus. Because our aim is showing that the upstream firm is able to extract the entire downstream surplus using this simple contract, there is no need to consider more complex schemes, including those that exclude some of the suppliers or also include a fixed fee to participate in the auction (as we did in the previous section).

We begin with assuming that, at the outset (i.e., before suppliers learn their cost) the upstream firm publicly offers to all n downstream firms a contract that consists of a fixed payment to be paid only if a supplier wins the auction. Call α such a payment and, without loss of generality, assume $0 \leq \alpha \leq v - \underline{c}$. Observe that it is an equilibrium for all suppliers to accept such contract ex-ante, as they can always guarantee for themselves a zero payoff by choosing their non-participation bid in the auction of the buyer.

In order to determine the contract that U will offer, we need to compute the equilibrium expected revenue of the upstream firm from such contract, for any given α and n. This requires, as a preliminary step, to solve for equilibrium both in the buyer optimal procurement auction and the efficient one, assuming suppliers operate under the specified contract with the upstream firm. Of course, while the efficient auction remains the same even when a contract between U and supplier is in place, our first step is to determine the format of the optimal auction for the buyer, taking into account the buyer observes the contract signed by suppliers and best-responds to it.This is what we do next.

Some more mechanism design. Recall that, for any equilibrium of any mechanism represented by the allocation rule $\boldsymbol{q}$, we let $Q_i(x) = \mathbb{E}_{c_{-i}}[q_i(x, c_{-i})]$ represent the interim allocation rule for i. Then, applying the envelope condition to the payoff of suppliers, we observe that the utility of a supplier i with cost c, must be equal to

$$\int_c^{\overline{c}} Q_i(x)dx$$

in any incentive compatible mechanism and, therefore, in both the efficient and optimal auction. Hence, the interim payment by the buyer to such type of supplier must be equal to

$$\int_{c}^{\overline{c}} Q_i(x)dx + Q_i(c)(c+\alpha),$$

where, as stated, α is the payment made by such bidder to U in case of winning the auction.

It follows that, in this new scenario, the buyer chooses an allocation rule $\boldsymbol{q}$ to maximize

$$\sum_i \mathbb{E}_{c_i}\left[vQ_i(c_i) - \int_{c}^{\overline{c}} Q_i(x)dx + Q_i(c_i)(c_i+\alpha)\right] =$$

$$\sum_i \left[\int_{\underline{c}}^{\overline{c}} Q_i(c_i)(v - c_i - \alpha)dG_i(c_i) - \int_{\underline{c}}^{\overline{c}}\int_{c}^{\overline{c}} Q_i(x)dx\ dG_i(c_i)\right] =$$

$$\sum_i \left[\int_{\underline{c}}^{\overline{c}} Q_i(c_i)\left(v - c_i - \alpha - \frac{G(c_i)}{g(c_i)}\right) dG_i(c_i)\right],$$

where the last equality is obtained after interchanging the order of integration for the second term in square brackets and then multiplying it and dividing it by g_i.

It becomes then apparent that the optimal auction is derived as usual, except now the virtual cost of a supplier with cost c becomes

$$\Gamma_\alpha(c) = c + \alpha + \frac{G(c)}{g(c)}.$$

That is, for the buyer, procuring from a supplier requires leaving the usual information rent plus covering the fixed cost α charged by the upstream firm.

Henceforth let $\hat{c}^o(\alpha)$ solve $\Gamma_\alpha(c) = v = \overline{c}$. Note that $\hat{c}^o(0) = \hat{c}$ and $\hat{c}^o(\overline{c} - \underline{c}) = \underline{c}$. Given symmetry, this is the reserve cost that defines the optimal second-price procurement auction in the same fashion of the previous section. Indeed, the optimal buyer auction is defined by the interim allocation rule

$$Q_i(c) = Q^o(c) = \begin{cases} (1-G(c))^{n-1} & \text{if } c \in [\underline{c}, \hat{c}(\alpha)] \\ 0 & \text{if } c \in (\hat{c}(\alpha), \overline{c}]. \end{cases}$$

Without buyer power, the auction remains as before. Except, now, a type whose cost is above $\hat{c}^e(\alpha) = \overline{c} - \alpha$ does not bid in the auction. (Note that $\hat{c}^e(\alpha) \geq \hat{c}^o(\alpha)$.) Hence

$$Q^e(c) = \begin{cases} (1-G(c))^{n-1} & \text{if } c \in [\underline{c}, \overline{c} - \alpha] \\ 0 & \text{if } c \in (\overline{c} - \alpha, \overline{c}]. \end{cases}$$

Upstream-optimal contract In light of the above, the revenue of the upstream firm in auction $a \in \{e, o\}$ with n bidders if it chooses contract α is

$$U_n^a(\alpha) = n\mathbb{E}_c\left[Q^a(c)\alpha\right] = \alpha \int_{\underline{c}}^{\hat{c}^a(\alpha)} n[1-G(c)]^{n-1} g(c) dc.$$

Let α_n^a maximize $U_n^a(\alpha)$ in α for $a \in \{e, o\}$. The main result of the section is the following.

Proposition 8 *Both α_n^a (> 0) and $U_n^a(\alpha_n^a)$ are increasing in n for $a \in \{e, o\}$. In the limit, as $n \to \infty$, the revenue of the upstream firm in both the efficient and the optimal auction converges to $v - \underline{c} = \overline{c} - \underline{c}$, that is $\lim_{n\to\infty} U_n^a(\alpha_n^a) = v - \underline{c}$ for $a \in \{e, o\}$.*

Proof We have $\alpha_n^a > 0$ since $U_n^a(0) = 0$ and $\frac{dU_n^a(\alpha)}{d\alpha}(0) > 0$. To see $U_n^a(\alpha_n^a)$ is strictly increasing in n for $a \in \{e, o\}$, we apply the envelope theorem

$$\frac{dU_n^a(\alpha_n^a)}{dn} = \frac{d}{dn}\left(n \int_{\underline{c}}^{\hat{c}^a(\alpha_n^a)} [1-G(c)]^{n-1} \alpha_n^a \; dG(c)\right) =$$

$$\alpha_n^a \int_{\underline{c}}^{\hat{c}^a(\alpha_n^a)} \frac{g(c)}{1-G(c)} \frac{\partial}{\partial n} \left(n[1-G(c)]^n\right) dc =$$

$$\alpha_n^a \int_{\underline{c}}^{\hat{c}^a(\alpha_n^a)} \frac{g(c)}{1-G(c)} \left([1-G(c)]^n + n[1-G(c)]^n \log(1-G(x))\right) dc =$$

$$\alpha_n^a \int_{\underline{c}}^{\hat{c}^a(\alpha_n^a)} g(c)[1-G(c)]^{n-1} \left(1 + n\log(1-G(x))\right) dc =$$

$$-[1-G(\hat{c}^a(\alpha_n^a))]^n \log(1-G(\hat{c}^a(\alpha_n^a)))\alpha_n^a.$$

and then observe that, since $\alpha_n^a > 0$,

$$-[1-G(\hat{c}^a(\alpha_n^a))]^n \log(1-G(\hat{c}^a(\alpha_n^a)))\alpha_n^a > 0$$

unless $G(\hat{c}^a(\alpha_n^a)) = 1$. The latter is the case only if $\alpha_n^a = v - \underline{c}$. But for finite n we must have $\alpha_n^a < v - \underline{c}$, or else the upstream firm obtains zero.

Next, let's look at monotonicity in the optimal choice of $\alpha \in [0, v - \underline{c}]$ (Recall once more $v = \overline{c}$). First note that

$$U_n^a(\alpha) = n \int_{\underline{c}}^{\hat{c}^a(\alpha)} [1-G(c)]^{n-1} \alpha \; dG(c) =$$

$$\int_{\underline{c}}^{\hat{c}^a(\alpha)} \alpha \; dG_{(1:n)}(c) =$$

$$\int_{\underline{c}}^{\overline{c}} \alpha \; \mathcal{I}_{\{c \le \hat{c}^a(\alpha)\}} \; dG_{(1:n)}(c).$$

Let $f(\alpha, c) = \alpha \mathcal{I}_{\{c \leq \hat{c}^a(\alpha)\}}$. Since $\hat{c}^a(\alpha)$ is decreasing in α for both $a = \{e, o\}$, then for any $\bar{c} \geq c \geq \hat{c}^a(0)$ we have $f(\alpha, c) = 0$. For $c < \hat{c}^a(0)$ we have $f(\alpha, c) = \alpha$ for $0 \leq \alpha \leq (\hat{c}^a)^{-1}(c)$ and zero otherwise. Note that $(\hat{c}^a)^{-1}(c)$ is decreasing in c, since $\hat{c}^a(\alpha)$ is decreasing in α. Next, we argue, for $c'' > c'$, $f(\cdot, c')$ dominates $f(\cdot, c'')$ in the interval dominance order defined in Strulovici and Quah (2009). To see this, observe that for any interval $[\alpha', \alpha'']$ such that $\alpha'' \in \arg\max f(\cdot, c'')$ and $\alpha' \notin \arg\max f(\cdot, c'')$ we have $\alpha'' \in \arg\max f(\cdot, c')$.

We conclude $\arg\max_\alpha U_n^a(\alpha)$ is increasing in n for $a = \{e, o\}$ by appealing to Theorem 2 in Strulovici and Quah (2009). To do so we note $G(\hat{c}^a(\alpha_n^a)) > 0$ and

$$\frac{g_{(1:n)}(c)}{g_{(1:n+1)}(c)} = \frac{n}{(n+1)(1 - G(c))}$$

is increasing in c, so that $g_{(1:n)}$ is a *monotone likelihood ratio shift* of $g_{(1:n+1)}$ for all $n \geq 1$.

Finally, we show that

$$\lim_{n\to\infty} \arg \max_{\alpha \in [0, v - \underline{c}]} U_n^a(\alpha) = v - \underline{c} \quad \text{for } a = \{e, o\}.$$

Notice that the limit exists as, for any n, $\arg\max_{\alpha \in [0, v-\underline{c}]} U_n^a(\alpha)$ is single-valued and, as shown above, is monotone increasing in n with upper bound $v - \underline{c}$.

For a fixed α, the revenue of the upstream firm is

$$\alpha \int_{\underline{c}}^{\hat{c}(a)} ng(c)[1 - G(c)]^{n-1} dc.$$

Then note that $ng(c)[1 - G(c)]^{n-1}$ is the density of the minimum of n draws. We know that for any given $z > \underline{c}$

$$\lim_{n\to\infty} \int_{\underline{c}}^{z} ng(c)[1 - G(c)]^{n-1} dc = 1.$$

So, $\lim_{n\to\infty} U_n^a(\alpha) = \alpha$.

To close the argument, notice $U_n^a(\alpha)$ is continuous in α and for any n the maximisation is taken over the compact set $[0, v - \underline{c}]$. So, by the maximum theorem, $\arg\max_{\alpha \in [0, v-\underline{c}]} U_n^a(\alpha)$ is continuous in n. Hence,

$$\begin{aligned}
\lim_{n\to\infty} \arg \max_{\alpha \in [0, v - \underline{c}]} U_n^a(\alpha) &= \arg \max_{\alpha \in [0, v - \underline{c}]} \lim_{n\to\infty} U_n^a(\alpha) \\
&= \arg \max_{\alpha \in [0, v - \underline{c}]} \alpha \\
&= v - \underline{c}
\end{aligned}$$

□

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