Endogenous Trading Networks*

Daniele Condorelli[†] Andrea Galeotti[‡]

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Abstract

We investigate the effects of a class of trading protocols on the architecture and efficiency properties of endogenously formed trading networks. In our model, the opportunity to sell valuable objects occurs randomly to different individuals. A sale can only be realized if two individuals are connected, directly or indirectly, but forming and maintaining a trading relation is a costly investment. When the outcome of trading is efficient and provides no intermediation rents, a tension between equilibrium and efficient networks emerges when the cost of forming a link is at an intermediate level. There are two types of inefficiencies. Either all equilibrium networks are under-connected when compared to efficient networks, or a multiplicity of equilibria may exist and agents may fail to coordinate on the efficient equilibrium network.

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[†]Department of Economics, University of Essex, dcond@essex.ac.uk.
[‡]Department of Economics, University of Essex, agaleo@essex.ac.uk.

1 Introduction

The majority of trade in goods and services takes place within a complex and fluctuating network of bilateral relationships, which are affected by bonds of trust developed through repeated interaction, ethnicity ties, geographical localization, and so forth.[1] We investigate the effects of a class of trading protocols on the architecture and efficiency properties of endogenously formed trading networks. For a set of efficient trading protocols that provide no intermediation rents, we analyze efficient and equilibrium trading networks in an economy where: the opportunity to sell valuable objects arises randomly to different individuals, a sale can only be realized if two individuals are connected directly or indirectly, forming and maintaining a trading relation is a costly investment.

In our model the events develop as follows. First, a trading network is formed and traders sustain the necessary cost. Second, a single indivisible object is randomly assigned to one of the traders. Then, individual values for the object are independently drawn. Values represent the maximum amount of money that each trader is willing to pay to consume the object. Finally, trade takes place. This framework captures an economy in which direct trade between two agents requires an investment (e.g., relationship, trust or infrastructure building) that is large with respect to the value of participating in a single trade.

We model trade in reduced form. A trading protocol assigns a unique payoff to each trader based on the network, the identity of the initial owner of the object, and the realized valuations. We focus on trading protocols that are ex-post efficient (i.e. the object is always consumed by the highest value trader in the component of the network where the object is allocated). In analyzing decentralized network formation, we restrict attention to trading protocols where intermediation rents are absent. That is, we consider trading protocols where the trade surplus is shared entirely (but in a rather unrestricted manner) between the initial owner of the object and the final buyer. Despite ex-post efficiency and absence of intermediation rents are strong properties, they emerge naturally in settings where, at the moment in which trade occurs, individual valuations for the object are common knowledge

[1]See Landa (1994), Greif (1993.) and Kranton (1996) for a variety of examples of the importance of personal connections for exchange. Macauley (1963) is a classic work on the relevance of trust in facilitating trade. Casella and Rauch (2002) provide examples on the importance of connections in international trade.

(e.g., see Gomes and Jehiel (2005), Gale and Kariv (2007), Kakade et al. (2004)).[2]

Decentralized network formation is modeled following Bala and Goyal (2000). We analyze Nash equilibria of a network formation game where agents independently and simultaneously decide which links to form, and pay a cost for each link that they form. Agents form their network before their values are realized and ownership of the object is assigned, but aware of how trading payoffs are determined as a function of these parameters and the network that is created.

Our first result characterizes efficient networks. We show that if a trading protocol is ex-post efficient, then ex-ante efficient networks are either minimally connected, when the cost of forming links is not too large, or empty (Proposition 1). We then show that when trading protocols are ex-post efficient and do not provide intermediation rents, equilibrium networks are either minimally connected or empty (Proposition 2).

These results illustrate that when the cost of forming links is either low or high, equilibrium networks are efficient. However, when the cost of forming links is intermediate, two types of inefficiencies may arise. First, it is possible that both the empty network and some minimally connected networks are equilibria, while the efficient network is minimally connected. In this case inefficiencies obtain when traders fail to coordinate on the good equilibrium. Second, we observe that for some levels of link cost the only equilibrium network is empty, while the efficient network is minimally connected. The equilibrium under-provision of trading links arises because traders do not fully internalize the social value of financing a link. Two features of our model contribute to this fact. First, two traders cannot contract on how to share the cost of a link that connects them. Second, the absence of intermediation rents prevents a trader from benefitting when other sellers exploits his connections to reach other potential buyers.

We conclude our analysis by studying an environment in which valuations are persistent. That is, the value of each trader is known before the network is formed. We show that if the trading protocol is efficient and such that the seller extracts all the surplus, then the conflict among efficient and equilibrium network disappears.

Our paper contributes to the emerging literature on decentralized trade in networks. Most

[2]Trading outcomes may not be ex-post efficient and intermediaries may earn profits in trading models where information is asymmetric (see Condorelli and Galeotti (2012))

of this literature has focused on understanding the outcome of trade in exogenously given networks (e.g., Corominas Bosh (2004), Condorelli and Galeotti (2012), Manea (2011), Abreu and Manea (2009), Elliot (2010), Nava (2009), Gale and Kariv (2007), Gale and Kariv (2009), Kakade et al. (2004), Blume et al. (2009)). Two exceptions are Kranton and Minehart (2001) and Elliot (2010) as both papers study the endogenous formation of buyer-seller networks.

Our paper also contributes to the large literature on network formation, see the seminal contributions of Bala and Goyal (2000) and Jackson and Wolinsky (1996).[3] Within this literature, a closely related paper is Goyal and Vega-Rendondo (2007). Their model can be interpreted as the representation of a trading protocol in which intermediation rents shared equally across intermediaries. Their main result is that that strategic link formation in such a setting leads to the star network.

Section 2 presents the model. The main analysis is developed in Section 3. Section 4 studies a model with persistent valuations. Section 5 concludes and discuss different interesting research questions for future work.

2 Model

Let $N = \{1, \ldots, n\}$ represent the set of traders that populate the economy. Traders are risk neutral, and maximize their monetary payoff. We assume that traders have "deep pocket" and are never budget constrained.

Actions in the economy develop as follows. First, a trading network is formed and traders sustain the necessary cost. Second, a single indivisible object is randomly assigned to one of the traders. We assume that each of the n traders becomes owner of the object with probability $1/n$. Then, individual values are independently drawn. Values represent the maximum amount of money that each trader is willing to pay to consume the object. Individual valuations for the object are independently drawn from a common distribution F. The distribution F has finite expectation and support on some positive open interval V of the real line. The distribution of ownership and the distributions of values are independent. Finally, trade takes place and then trading payoffs are realized.

[3] The literature is too vast to be summarized here and we refer to Goyal (2007) and Jackson (2008) for a survey treatment.

3 Analysis

We start backwards by discussing the trading stage. We then develop our analysis of network formation and provide the main results of the paper.

3.1 Trading Stage

Before trade takes place, the network is formed, the object is assigned and values are drawn. We denote s the agent who has been assigned ownership of the object and we let $\boldsymbol{v} = (v_1, \ldots, v_n)$ indicate the profile of monetary values. Abusing notation, we define a network G as set of edges among the traders in N. We say that i and j have a trading link if ij belongs to G. We restrict attention to undirected networks.[4] In addition, we define a component C_x as a subset of N consisting of traders that are all connected directly or indirectly among themselves.[5]

We model trade in reduced form. A *trading protocol* is a mapping that assigns a unique payoff to each trader as a function of the network, the realized valuations, and the owner of the object. Let C_s be the component to which the initial owner s belongs. A trading protocol satisfies the following definition.

Definition 1. *A* trading protocol Y *is a set of non-negative and real-valued functions* $Y_1(s, G, \boldsymbol{v}), \ldots, Y_n(s, G, \boldsymbol{v})$ *such that, for all* $(s, G, \boldsymbol{v})$*, we have (i)* $Y_i(s, G, \boldsymbol{v}) = 0$ *for all* $i \notin C_s$ *and (ii)* $\sum_{i \in C_s} Y_i(s, G, \boldsymbol{v}) \leq \max_{j \in C_s} v_j$.

Condition (i) and condition (ii) are natural and follow from the idea that trade between two agents requires either a direct link between them or that the two agents are connected indirectly. Hence, the trading payoff must be zero for all traders who are not directly or indirectly linked to the seller, and, for the traders in C_s, it cannot overall exceed the maximum consumption value in C_s.

[4] That is we consider networks such that for any $i, j \in N$ if $ij \in G$ then also $ji \in G$.

[5] We say that two traders are directly or indirectly connected if there is a path among them. There is a path between i and j if either $ij \in G$ or there exists $\{j_1, \ldots, j_k\} \in N$ such that $\{ij_1, j_1j_2, \ldots, j_kj\} \in G$. Therefore, a component C_x is a subset of N such that there is a path between every $i, j \in C_x$ and there is no path between every $i \in C_x$ and $j \notin C_x$

Different assumptions on Y describe the outcome of different trading protocols. In this paper we focus on the class of ex-post efficient trading protocols.

Definition 2. *A trading protocol is ex-post efficient if, for all* $(s, G, \boldsymbol{v})$*, we have* $Y_j(s, G, \boldsymbol{v}) = 0$ *for all* $j \notin C_s$*, and*

$$\sum_{i \in C_s} Y_i(s, G, \boldsymbol{v}) = \max_{i \in C_s} v_i.$$

In an ex-post efficient trading protocol the sum of utility from trade obtained by the agents in the component where the object has been allocated equals the maximum willingness to pay across agents in that component. So, all potential surplus within a component is realized. Observe that the definition above also implies that there is no waste of resources in trading. That is, trading is frictionless.[6]

Next, to complete the characterization of the set of trading protocols that we analyze, we introduce an additional restriction. This concerns the way that the trading surplus is shared among the members of the component to which the seller belongs. For a given $\boldsymbol{v}$, denote by $b = \arg\max_{i \in C_s} v_i$ the agent in C_s with the highest value and with $\boldsymbol{v}_{C_s \setminus \{s,b\}}$ the un-ordered vector of values of the traders in C_s with the exclusion s and b.

Definition 3. *An ex-post efficient trading protocol provides no intermediation rents if there exists a function* $\alpha : \{V^2 \times \{\emptyset, V, V^2, \ldots V^{n-2}\}\} \to [0, 1]$ *such that, for all* $(s, G, \boldsymbol{v})$

$$Y_s(s, G, \boldsymbol{v}) = \alpha(v_s, v_b, \boldsymbol{v}_{C_s \setminus \{s,b\}}) v_b = v_b - Y_b(s, G, \boldsymbol{v}).$$

In an ex-post efficient trading protocol that provides no intermediation rents, the entire surplus is shared only between the seller and the final buyer. All other traders remain at zero utility. The way in which the surplus is shared between s and b may depend on the realized values within the component to which the seller belong, but not on the identities of the traders.

While the restrictions imposed above are not innocuous, they emerge naturally in a wide variety of contexts where, at the moment in which trade occurs, the valuations for the object of each trader are common knowledge (see for example Gale and Kariv (2007), Gale and Kariv (2009), Kakade et al. (2004)).[7] However, as showed by Condorelli and Galeotti (2011),

[6] Two typical frictions in these environments might be transaction costs and discounting.

[7] We refer to Gomes and Jehiel (2005) for a general account of why dynamic process of social interaction with complete information and no externalities tend to land into efficient outcomes.

intermediaries may earn a rent and trading outcomes may not be efficient if information is incomplete.

We provide here three examples of ex-post efficient trading protocols with no intermediation rents.

Example 1 (Seller's monopoly). *Let $\alpha = 1$. In this case the designated initial owner of the object extracts all the surplus. The buyer is left with zero utility. This outcomes arise naturally in an environment where sellers have full bargaining power.*

For example, consider the following finite-horizon dynamic game with complete information. Each round of trade $t = 1, ..., T$, with $T > n - 1$, develops in three steps. First, the owner of the object at the beginning of round t, say i, makes a take-it-or-leave-it offer to one of his neighbors, say j. Second, trader j either accepts or rejects the offer. If j accepts, i transfers the good to j and j pays the agreed price, and we move to the third stage. If trader j rejects, we move to the third stage. Third, the current owner, either i or j, decides whether to consume the object or not. If the owner consumes the object the game ends. Otherwise the game moves to the subsequent round of trader $t+1$. The game also ends when it reaches the end of the last round of trade T. In the game above, when T is sufficiently large, the unique subgame perfect equilibrium outcome is Pareto efficient and the initial owner of the object extracts all the surplus(see Condorelli and Galeotti (2011)).[8]

Example 2 (Competitive equilibrium). *Let $\alpha = \max_{i \in G_s/b} v_i/v_b$. In this case the seller obtains a profit equal to the second highest value in the component while the buyer collects the difference between his value and the second highest value. Observe that the second highest value is the minimum price for the object that is compatible with market clearing within the component.*

Example 3 (Buyer's market). *Let $\alpha = \frac{v_s}{v_b}$. In this case the seller makes zero profit and the buyer extracts the difference between his value and the value of the seller.*

The requirement that the trading protocol is ex-post efficient and provides no intermediation rents has sharp consequences on traders'payoffs. Let $U_i(G)$ indicate the utility that trader i expects from trading in network G *prior* to the realization of values and to the allo-

[8]The same outcome would be also achieved if the seller were constrained to use second-price or English auctions with endogenously chosen reserve price, rather than take-it-or-leave-it offers.

cation of ownership of the object. Let $W(G) = \sum_{i \in N} U_i(G)$ represent the ex-ante utilitarian welfare generated by trading in network G.

Let $v_{1:n} \leq v_{2:n} \leq \cdots \leq v_{n:n}$ denote the order statistics from a sample of n independent random variables all identically distributed with distribution F. Let $E[v_{k:k}] = \mu_k$ be the expected value of the maximum of k draws. Furthermore, for a component C_x of G, let g_x indicates the number of traders in C_x.

Lemma 1. *Consider a network G with components $C_1, \ldots, C_l$.*

1. *If the trading protocol is ex-post efficient then*
$$W(G) = \sum_x \frac{g_x}{n} \mu_{g_x}. \tag{1}$$

2. *If the trading protocol is ex-post efficient and provides no intermediation rents then for any C_x and $i \in C_x$*
$$U_i(G) = \frac{1}{n} \mu_{g_x}. \tag{2}$$

Proof. Suppose the trading protocol is ex-post efficient. If the object lands to component C_x, then the expected welfare generated is μ_{g_x}. Since the object is allocated randomly, the probability that the object is allocated to an agent belonging to C_x is $1/g_x$. Combining we obtain expression 1.

Suppose now that the trading protocol is ex-post efficient and it provides no intermediation rents. Consider a trader $i \in C_x$. The probability of becoming the owner is $1/n$, the probability of the object being assigned to another member of the component is $(g_x - 1)/n$, and the probability that $i = b$ conditional on the object being owned by someone in C_x is $1/g_x$. Therefore, the ex-ante expected payoffs to agent $i \in C_x$ is

$$U_i(G) = \frac{1}{n} \left(\frac{1}{g_x} \mu_{g_x} + \frac{g_x - 1}{g_x} E\left[\alpha(v_s, v_b, \boldsymbol{v}_{\{G_s/b,s\}}) v_b \mid s = i, b \neq i \right] \right) +$$
$$+ \frac{n-1}{n} \frac{g_x - 1}{n-1} \frac{1}{g_x} E\left[v_b - \alpha(v_s, v_b, \boldsymbol{v}_{\{G_s/b,s\}}) v_b \mid s \neq i, i = b \right] = \frac{1}{n} \mu_{g_x},$$

since $E\left[\alpha(v_s, v_b, \boldsymbol{v}_{\{G_s/b,s\}}) v_b \mid s \neq i, i = b \right] = E\left[\alpha(v_s, v_b, \boldsymbol{v}_{\{G_s/b,s\}}) v_b \mid s = i, b \neq i \right]$. ■

In words, whenever the bargaining protocol is efficient and provides no rent to intermediaries, the ex-ante expected payoff of a trader, and hence his incentives at the network formation stage, are determined solely by the number of traders that belong to his component. In particular, symmetry considerations imply that each trader in a component shares the same fraction of the total expected surplus that is generated by trading in that component.

3.2 Network Formation

This section presents our main results. We first study ex-ante efficient networks and then we characterize Nash equilibrium networks. We finally discuss the inherent conflict between efficiency and equilibrium that results from the analysis.

Some further definitions are required. A network G is connected if all traders are directly or indirectly connected (i.e., the network has only one component). A network is minimal if removing an arbitrary trading link increases the number of network components. Denote by η the total number of *undirected* trading links in G. A trader is isolated if he is not connected to any other trader. A network is empty if every trader is isolated.

3.2.1 Efficient networks

We assume that each undirected link has a cost c and we focus on trading protocols that are ex-post Pareto efficient. The problem of the social planner is to construct a trading network that maximizes the ex-ante social welfare net of the trading link costs. In view of the first part of Lemma 1, the problem of the social planner is to maximize in G the following:

$$\sum_x \frac{g_x}{n} \mu_{g_x} - \eta c.$$

We call ex-ante efficient, or simply *efficient*, a network that maximizes the above objective function.

In order to proceed with the analysis we introduce a restriction on distribution of values that is instrumental in formulating our next result.

Assumption 1. *The distribution F is such that for every $k = 3, \ldots, n$:*

$$k\mu_k - (k-1)\mu_{k-1} > (k-1)\mu_{k-1} - (k-2)\mu_{k-2}.$$

Assumption 1 states that the social value from adding one more trader to k other traders (i.e.,the extra value to the trader plus the value to other traders) is increasing in k. This means that the multiplier effect of many people benefitting from the extra node exceeds the reduction in extra surplus that comes by the fact the the highest value in the component increases but in a concave fashion. Assumption 1 is verified in a wide range of circumstances. For example assumption 1 holds in the case of the uniform distribution, the family of power distributions and the family of exponential distributions.

Proposition 1. *Assume that the trading protocol is ex-post efficient. An efficient network is minimal. If Assumption 1 holds then:*

1. *if* $c \leq \frac{1}{n-1}(\mu_n - \mu_1)$ *an efficient network is minimally connected*

2. *if* $c \geq \frac{1}{n-1}(\mu_n - \mu_1)$ *the efficient network is the empty network.*

Proof. An efficient network must be minimal. Assume now that Assumption 1 holds. Take a minimal network G and assume that there are at least two non-singleton components, say C_j and C_k and recall that g_j and g_k represent the number of nodes in the two components. Assume, without loss of generality, that $g_k \geq g_j$. Consider the change in welfare that can be obtained by removing an extremal trader from the smaller component C_j and adding the trader to the larger component C_k. Note that the new network has the same number of links as the original one. The shift of the trader is welfare beneficial if, and only if,

$$k\mu_k + j\mu_j < (k+1)\mu_{k+1} + (j-1)\mu_{j-1}.$$

Rewriting, the change is beneficial if, and only if,

$$j\mu_j - (j-1)\mu_{j-1} < (k+1)\mu_{k+1} - k\mu_k,$$

which holds from assumption 1.

Hence, if G is efficient it has to be the case that there is a non-singleton component C_k with size $g_k \in \{0, \ldots, n\}$ and if $j \notin C_k$ then j has no links, i.e., j is isolated. So, consider such a network G. We show that either we want to disconnect one trader from C_k and make him isolated or we want to connect an isolated trader to C_k.

Consider the change in welfare from disconnecting one player from C_k. This is positive if, and only if,

$$(k-1)\mu_{k-1} + (n-k+1)\mu_1 > k\mu_k + (n-k)\mu_1 - nc,$$

or, equivalently,

$$c > \frac{1}{n}\left[k\mu_k - (k-1)\mu_{k-1} - \mu_1\right].$$

Then, note that adding an isolated node to C_k is beneficial if, and only if,

$$(k+1)\mu_{k+1} + (n-k-1)\mu_1 - nc > k\mu_k + (n-k)\mu_1,$$

or, equivalently,

$$c < \frac{1}{n}\left[(k+1)\mu_{k+1} - k\mu_k - \mu_1\right].$$

Since, by assumption 1, $(k+1)\mu_{k+1} - k\mu_k > k\mu_k - (k-1)\mu_{k-1}$, it follows that it is always welfare improving to either removing one node from C_k or adding one isolated player to C_k. This holds for any k. Hence, an efficient network is either empty or minimally connected.

To complete the proof it is sufficient to observe that when $\frac{1}{n-1}(\mu_n - \mu_1) > c$, the welfare of the minimally connected network is above the welfare of the empty network. The opposite happens if $\frac{1}{n-1}(\mu_n - \mu_1) < c$. ■

As the number of traders in the economy grows large the efficient network is necessarily empty. There are two ways of looking at this result. First, the gain from connecting an extra trader are decreasing as the number of traders grows. Hence, for any fixed c, as the number of traders grows, the connected network becomes necessarily inefficient. This result is analogous to the result that the efficiency gains from having an extra participant to an auction are decreasing in the number of individuals that participate. Second, as the number of traders grows large the social value of a single connection is decreasing. Given the random ownership allocation process, when the number of traders grows the possibility that the link is actually useful for performing trade shrinks.

3.2.2 Decentralized network formation.

In this section we consider a game of unilateral network formation, following Bala and Goyal (2000). Briefly, at the beginning of the game, each agent announces links to other agents.

An undirected link between i and j is formed if at least one of the two agents announce a link to the other. If in network G agent i announces η_i links agent i pays a total linking cost of $\eta_i c$. We focus on trading protocols which are ex-post efficient and in which there is no intermediation rent.

In view of part 2 of Lemma 1, the ex-ante expected utility of agent i who belongs to component C_x is

$$\frac{1}{n}\mu_{g_x} - \eta_i c, \tag{3}$$

where, we recall, g_x is the number of traders in C_x.

It is immediate to observe that, regardless of the distribution of surplus among buyer and seller, keeping his connection costs fixed, a trader will always prefer to be connected to a larger rather than smaller component of the network. This implies that a network with two components each containing more than one trader cannot be equilibrium. We obtain the following characterization result.

Proposition 2. *Consider ex-post efficient trading protocols that provide no intermediation rents. A Nash equilibrium network is either empty or minimally connected.*

1. *The exists a minimally connected equilibrium network* if and only if

$$c \leq \frac{1}{n}(\mu_n - \mu_1).$$

2. *The empty network is equilibrium* if and only if

$$c \geq \frac{1}{n}(\mu_2 - \mu_1).$$

Proof. The first part of the proposition follows as a corollary of Proposition 4.1. of Bala and Goyal (2000). Indeed, note that since μ_{g_x} is increasing in g_x, expression 3 satisfies the assumptions of Proposition 4.1 in Bala and Goyal (2000).

We now show that there exists a minimally connected equilibrium network if and only if $c \leq \frac{1}{n}(\mu_n - \mu_1)$. To prove the if part, take a network where each agent $\{1, \ldots, n-1\}$ announces a link to agent n, and there are no other links. Every agent $i \neq n$ obtains $\frac{1}{n}\mu_n - c$, which, under the stated condition, is higher than the utility that i obtains by deleting the link with the center, which is $\frac{1}{n}\mu_1$. To prove the only-if part note that if $c > \frac{1}{n}(\mu_n - \mu_1)$, then a trader who finances one or more links in a minimally connected network would be better off by severing his links. This is true because: the benefit from each link is at most μ_n/n, the marginal cost of each link is c and the minimum payoff that an agent obtains without sponsoring links is μ_1/n.

We now show that the empty network is equilibrium if, and only if, $c \geq \frac{1}{n}(\mu_2 - \mu_1)$. If $c < \frac{1}{n}(\mu_2 - \mu_1)$ the empty network is not an equilibrium, as the payoff from financing a link with someone who is also isolated is strictly positive. Then, suppose that $c \geq \frac{1}{n}(\mu_2 - \mu_1)$, and for a contradiction assume that the empty network is not an equilibrium. By definition, this implies that, starting from being isolated, an agent must gain from forming $1 \leq k \leq n-1$ links to different isolated traders. Therefore, it must be the case that, for some k:

$$\frac{1}{n}\mu_{k+1} - kc > \frac{1}{n}\mu_1.$$

Rewriting, we must have:

$$\frac{1}{kn}(\mu_{k+1} - \mu_1) > c.$$

Since $c \geq \frac{1}{n}(\mu_2 - \mu_1)$, we obtain that:

$$\frac{1}{k}(\mu_{k+1} - \mu_1) > (\mu_2 - \mu_1),$$

which is impossible because $\mu_{k+1} - \mu_k < \mu_k - \mu_{k-1}$ for any $k = 1, \ldots, n-1$ (i.e., the expected maximum of k draws is increasing in concave in k). ■

We observe that all trading protocols which are ex-post efficient and provides no intermediation rent lead to a payoff specification which is consistent with the Bala and Goyal (2000) specification. In this sense, the model we develop provides a natural and economically interesting example of Bala and Goyal (2000).

Finally note that, when the cost of forming a link is moderate, there is a multiplicity of equilibria: both the empty network and a set of different minimally connected networks are equilibria. We also note that every minimally connected network can be sustained as an equilibrium for certain ranges of c. However, the minimally connected network which is equilibrium for the widest range of cost levels is the periphery sponsored star network (i.e., a network where one agent sponsors no links, the center, and all other agents sponsor a single link to the central player). In a periphery sponsored star, the removal of a link

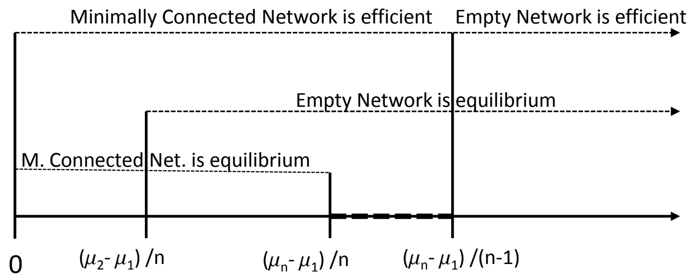

Figure 1: Equilibrium vs Efficiency

by a peripheral trader will preclude him any further negotiation with all other traders, and therefore the cost of removing such a link is (weakly) higher than the cost of removing any other link in any other network.

3.3 Conflict between efficiency and equilibrium

Comparing proposition 1 and proposition 2, we can evaluate whether individual incentives are aligned to social interest. Figure 1 summarizes the two propositions and illustrates the conflict between efficiency and equilibrium.

Our results show that when the cost of forming a link is high or low, decentralized network formation generates efficient networks. However, we obtain a different result when the cost of forming and maintaining an edge is at an intermediate level. There are two types of inefficiencies, which we now discuss.

First, when the cost of a link is $c \in \left(\frac{1}{n}\left(\mu_n - \mu_1\right), \frac{1}{n-1}\left(\mu_n - \mu_1\right)\right)$ (see dotted region in the picture), the only equilibrium outcome is the empty network, while the efficient network is non-empty. So, the equilibrium network is under connected as compared to the socially efficient trading network. Note that such inefficiency vanishes as the number of traders grows large. For example, when values are drawn from a uniform distribution with values in $[0, 1]$ inefficiency is present when $c \in \left(\frac{n-1}{2n(n+1)}, \frac{n-1}{2(n^2-1)}\right)$.

The under-provision of trading links in the equilibrium of our decentralized network formation game is a natural consequence of the fact traders do not fully internalize the social value of financing a link. This fact can be, at least partially, attributed to two features of our model. First, traders cannot share the cost of link. This feature prompts inefficiencies even in the two traders case. When i contemplates forming a link with j he will not take into account that j will also benefit from the link. Second, there are no intermediation rents. Under this assumption, when trader i evaluates a link to trader j, he does not internalize that the investment in the new link will also be beneficial to other agents every time trader i will be used as intermediary via that new link.

Second, consider intermediate cost levels in the interval $\left(\frac{1}{n}\left(\mu_2-\mu_1\right), \frac{1}{n}\left(\mu_n-\mu_1\right)\right)$. In this case there is multiplicity of equilibria: the empty network is always an equilibrium and, for each cost level in this region, there is always a minimally connected equilibrium network. The efficient network is minimally connected. In this case inefficiencies may emerge because agents coordinate to a bad equilibrium. The idea here is that agents must expect that other traders invest enough in the trading network to justify their own investment in new links.

To conclude, we provide a further comment on the efficiency property of the periphery sponsored star network. In a periphery sponsored star network, the removal of a link by a peripheral trader i precludes him from any further negotiation with other agents and also minimizes the disruption to other traders, which are only precluded from trading with trader i. In other words, in a periphery-sponsored star all connections have the same social value, and this is, for the largest part, represented by the private value to the agent who is sponsoring the link. Since, in expectation, the equilibrium payoff obtained by trading is the same across traders, the periphery sponsored star architecture minimizes, across all minimally connected networks, the maximal difference between the private and social value of a link.

4 Persistent Valuations

In this section we modify the model previously introduced in order to capture a scenario in which values are persistent. We assume that the values are drawn and become common knowledge *before* the network is formed. The rest of the model is unaltered. We can interpret this scenario as one in which the relative valuation ranking among the traders is known in advance but sales opportunities arise quickly and still make the network persistent. We shall show that in this environment, there is no trade-off between efficiency and equilibrium, as long as the trading protocol is ex-post efficient and has the property that the seller extracts all the surplus. Since the vector of valuations is common knowledge before the network is formed, we can, without loss of generality, assume that $v_1 < v_2 < \cdot < v_n$.

We first characterize the social planner problem. The social planner chooses the network G that maximizes the sum of the expected utility of each trader, net of the cost of maintaining the links. Under an ex-post efficient trading protocol the total welfare produced by the network is:

$$\sum_j \frac{g_j}{n} \max_{i \in C_j} v_i - \eta c.$$

Let $L(c) = \left\{ j \in N : \frac{v_n - v_j}{n} \geq c \right\}$. In words, $L(c)$ is the set of traders for which the value is low enough that the expected net profit from selling to n (the highest value trader) exceeds the cost c of maintaining the link.

Proposition 3. *Consider an ex-post efficient trading protocol. Under persistent valuations, the network G is efficient if and only if the traders in $L(c) \cup \{n\}$ belong to a minimally connected component of G and all the other traders are isolated.*

Proof. The efficient network must be minimal. Next, we claim that every trader $j \in L(c)$ must be in the same component of trader n. For a contradiction suppose $j \in L(c)$ does not belong to the same component of trader n. If j is isolated then forming a link between j and n creates an expected social benefit of $(v_n - v_j)/n$ and a social cost of c. Because $j \in L(c)$ creating the link increases total welfare. Suppose j is not isolated. Let $j \in C_j$ and $n \in C_n$, and let $v^* = \max_{i \in C_j} v_i$. The welfare generated by G is

$$SW(G) = \frac{g_j}{n} v^* + \frac{g_n}{n} v_n + A - \eta c,$$

where A is some constant produced by the other components. Consider now the network G' derived by G as follows: first delete all the links in C_j, then add a link between n and every $i \in C_j$ with the exception of the highest value trader in C_j. Note that the new network has the same number of links of G. We get:

$$SW(G') = \frac{1}{n} v^* + \frac{g_j - 1}{n} v_n + \frac{g_n}{n} v_n + A - \eta c,$$

and therefore $SW(G') > SW(G)$ as long as $v^* < v_n$. So, every trader $j \in L(c)$ must access trader n. It is immediate to see that every trader $j \notin L(c) \cup n$, must be isolated. This completes the proof of the proposition. ■

Observe that the social planner would like to connect the highest value trader to all low value traders, while maintaining isolated average value traders for which the gains from trade, and hence the gains from being connected, are lowest.

Proposition 4. *Consider a trading protocol that is ex-post efficient, provides no intermediation rents and all the surplus from trade goes to the seller, i.e.,* $\alpha = 1$. *Every Nash equilibrium network is ex-ante efficient. Conversely, an ex-ante efficient network is a Nash equilibrium if: agent* i *sponsors a link to agent* j *if and only if agent* j *belongs to the path connecting agent* i *to the highest value agent* n.

Proof. Let G be a Nash equilibrium. Clearly G must be minimal. Consider an agent i who sponsors a link to j. If i does not access n via j, then i will strictly prefer to delete the link with j and either form no link or form a link with trader n. So, if i sponsors a link to j, agent i must access n via j. Since G is minimal, this implies that each agent forms at most one link, and n forms no link. It is immediate to see that j will access n in equilibrium if and only $j \in L(c)$. Hence G is efficient. We have also already proved the converse. ■

Therefore, when valuations are persistent, an efficient trading protocol in which the initial owner extracts all the surplus aligns individual incentives to the incentives of a social planner.

5 Discussion

We analyzed a model in which the opportunity to sell valuable objects occur randomly to different traders, a sale can only be realized if two individuals are directly or indirectly connected, and maintaining a trading relation requires costly investments. We characterize efficient networks for trading protocols that are ex-post efficient and satisfy a mild technical condition. We characterize equilibrium networks under the additional restriction that there are no intermediation rents.

The analysis provides insight on the trade-off between efficient networks and equilibrium networks. In particular endogenous networks may be inefficient, in some range of the parameters values, for two reasons. First, because traders may fail to coordinate into a good equilibrium. Second, because they do not internalize all the benefits that a connection generates. We observe that the latter effect might be mitigated if cost sharing of links was possible or intermediation rents were allowed. However it remains a question open for future research whether, even possibly allowing for link cost-sharing, there exist a trading protocol with intermediation rents, that induces efficient equilibrium outcomes.

The question is not trivial since when there are intermediation rents traders may be tempted to over invest in connections in order to break the intermediation powers of other trader. This effect is illustrated, for example, in Condorelli and Galeotti (2012).[9]

Finally, we have considered that only one object is traded in the network. All the insights could be extended easily to multiple independent objects. The study of the formation of trading networks when the trade involves multiple objects who are either substitutes or complements is an open question for future research.

[9]Another example where intermediation rents may provide agents the incentives to create over connected networks is Goyal and Vega-Redondo (2007).

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