Selling through referrals

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Abstract
We endogenize intermediaries’ choice to operate as agents or merchants in a market where there are frictions due to asymmetric information about consumption values. A seller has an object for sale and can reach buyers only through intermediaries. Intermediaries can either mediate the transaction by buying and reselling—the merchant mode—or refer buyers to the seller for a fee—the referral mode. When the seller has a strong bargaining position and can condition the asking price to the intermediaries’ business model choice, all intermediaries specialize in agency. The seller’s and intermediaries’ joint profits equal the seller’s profits when he has access to all buyers. When the seller does not have such bargaining power, the level of the referral fee and the degree of competition among intermediaries determine the business mode adoption. A hybrid agency–merchant mode may be adopted in equilibrium. Banning the referral mode can decrease welfare because the merchant mode is associated with additional allocative distortions due to asymmetric information.

1 | INTRODUCTION

An intermediary is an economic agent that facilitates the connection between supply and demand. Intermediaries purchase from suppliers and resell to customers or facilitate the meeting of buyers and sellers. For example, online travel agents such as Expedia, Priceline, and Orbitz connect buyers to hoteliers, airlines, and car rental companies. Amazon marketplace, Groupon, and alike, connect buyers to retailers selling a variety of goods. Amazon and Apple intermediate the E-books industry. Intermediation is also important in off-line markets and beyond traditional retailing. For instance, it plays a key role in trading real estate, art, used cars, books, as well as in markets for professional services.

Intermediaries predominantly operate under two business models — or hybrids of those. Under the referral mode (or agency mode), they refer buyers to sellers, who then negotiate directly on the terms of trade. In return, intermediaries receive referral fees for the creation of the match and/or commissions based on sales. Under the more traditional retailer/merchant mode, intermediaries buy goods from suppliers for resale to consumers.

In the online travel industry, Priceline makes most of its revenue through the referral mode and the rest from acting as a merchant. Priceline’s subsidiary Booking.com is an agency-based business, whereas its subsidiary Agoda is a merchant-based business.2 Expedia operates mainly under the merchant mode receiving roughly 75 percent of its revenue through the merchant mode.
mode and some 21 percent through the referral mode.\textsuperscript{3} Expedia expanded its business by acquiring the agency-based online hotel business Venere. Orbitz’s net revenue stems fairly evenly from its air and hotel businesses (34 and 29 percent respectively), with its revenue from the hotel business coming mainly from the merchant mode.\textsuperscript{4}

In this paper, we explicitly model the choice of an intermediary to operate as a merchant or a reseller.\textsuperscript{5} Beyond providing insights on the equilibrium structure of the industry, we also analyze the effects of the two models on market outcomes and welfare. Finally, we draw some implications for competition policy. Crucially, we consider markets where both options, to refer or to resell, are available to intermediaries and not controlled by sellers.

An intermediary who chooses its business model faces the following trade-off, which is at the core of our analysis. By operating as an agent, the intermediary’s revenue only depends on the number of consumers he refers to the seller and on the referral fee. There is no uncertainty, nor a dependence on the market structure. Instead, by becoming a merchant, the intermediary makes a risky profit that depends on the difference between the expected cost and revenue of buying and reselling. These variables, in turn, depend on the market structure, not just on the number of buyers, and on the protocols that regulate trade upstream and downstream.

We develop a simple model that captures some of the forces shaping intermediaries’ decisions to operate as merchants or agents, and allows us to perform comparative statics. In our baseline model, there is a seller with one object for sale. The seller is in contact with a number of intermediaries, each intermediary has access to a subset of buyers, and each buyer has private information about his valuation. The seller and the intermediaries have no consumption value for the object. The interaction between the seller, the intermediaries, and the buyers is captured by a three-phase game. In the business-model choice phase, each intermediary decides whether to become an agent or a merchant. An agent-intermediary refers all her buyers to the seller in exchange for a referral payment. In the trading phase, the choices of intermediaries become public, the seller sells to referred buyers and merchant intermediaries. If a referred buyer obtains the object, the game ends. If a merchant intermediary obtains the object, then we enter the resale phase: the merchant intermediary resells to the buyers he is in contact with.

Our model captures salient features of several industries. One notable example is the market for hotel rooms. First, hotel rooms are limited in number, and supply cannot be increased in the short term. Second, hotel rooms are extremely differentiated (by city area, comfort level, etc.) and therefore their sellers may enjoy nonnegligible market power. Third, multiple intermediaries operate in the market and both intermediaries and sellers can potentially conduct the sale. As we have mentioned earlier, both merchant intermediaries, such as Expedia, and agents, such as Booking, operate in the market. Finally, as in our model, it is costly for consumers to look for hotels without shopping via an intermediary, and likewise hotels have no mean to directly advertise to potential buyers in a cost-effective way.\textsuperscript{6}

Our first result (Theorem 1) shows that when the seller has full bargaining power and can choose the selling procedure, regardless of the level of referral payments, in the unique equilibrium of the game intermediaries adopt the referral model. The prevalence of the referral mode is a consequence of the seller’s ability to tailor the minimum price to the intermediary’s business model choice. Indeed, the seller anticipating the resale value of a merchant intermediary never sets the minimum reserve price below that value. Thus, merchants enjoy no rent from buying and reselling. As a consequence, adopting the referral mode is strictly preferable.

Because referrals operate frictionlessly, the industry’s specialization in agency implies that the aggregate equilibrium profit of both the seller and the intermediaries equals the expected seller’s profit when he has direct access to all buyers—what we call the “integrated-industry profit.” Hence, in equilibrium mis-allocation distortions arising from intermediation via resale are eliminated. In particular, there is no risk that the “wrong” intermediary acquires the object and resells it to a low-value buyer, thereby excluding out of the market high-value buyers who are clients of different intermediaries.

When all intermediaries adopt the referral mode, not only industry profit but welfare, including consumer surplus, is unambiguously higher than in the case in which some intermediaries adopt the merchant mode. This follows from the observation in the previous paragraph and from the fact that both the seller and a merchant intermediary choose the same reserve price.\textsuperscript{7}

This finding contributes to the industrial-organization literature on vertical restraints and provides additional insights into a policy-relevant issue.\textsuperscript{8} More precisely, we provide a market-based rationale for the adoption of a referral mode and show that, ultimately, consumers may benefit from such a vertical agreement. Because higher prices in our setup reflect a more efficient allocation, buyers may benefit even in cases in which the final price ends up being higher than under the merchant mode. This observation could be relevant for competition policy in view of the increasing number of intermediaries that operate as agents, and the new challenges that intermediated markets pose for competition authorities.

As a concrete example, consider the United States v. Apple Inc. case. In 2012 the U.S. Department of Justice (DOJ) filed a lawsuit against Apple and a number of major publishers for an alleged conspiracy, having the objective to raise e-book prices (see Gaudin & White, 2014). At the core of the case is a shift of the e-book industry toward the referral mode. Whereas before
Apple's entry into the market the price of e-books was set by resellers (such as Amazon), after Apple's entry into the market the industry converged to a new standard, characterized by the referral mode. The DOJ claimed that a shift to a referral mode, prompted by Apple, played an instrumental role in a collusive agreement between publishers and Apple with the aim of raising e-book prices. Although not providing an explicit counterargument to the court's finding, our analysis suggests a channel through which consumers may have benefited from the adoption of the referral mode, despite the alleged increase in e-book prices.

The result that the referral mode emerges as the unique equilibrium outcome is not limited to the case in which the seller has full bargaining power but generalizes widely to other trading environments where the seller is able to condition his pricing strategy on the intermediaries' decisions. In Appendix B, we show that Theorem 1 holds under much weaker assumptions: The seller can employ other selling protocols, intermediaries can have access to different numbers of buyers who belong to different subgroups, and can have private information both about which buyers they know as well as about these buyers' valuations.

In some cases, however, the seller cannot fully condition his pricing strategy to the business choices of the intermediaries. This might be the case, for instance, because of competition among sellers in the upstream market that drives price down to cost and limits the pricing flexibility of sellers. Or, it may be too costly for the seller to observe and react to the choice of intermediaries to refer or not. To address this case, in the second part of the paper we assume the seller auctions the object either without a minimum reserve price or at a reserve price that is fixed and does not depend on whether or not intermediaries are among the buyers.

The predictions change substantially. When the referral fees are sufficiently high, intermediaries still adopt the referral mode. However, when referral fees are low, intermediaries adopt a hybrid business model, that involves both agency and merchant activities. In particular, in the only symmetric equilibrium, intermediaries randomize between the two business models (Theorem 2). As expected, in this symmetric equilibrium, the merchant mode arises with higher probability the smaller the referral fee. In contrast, when the number of intermediaries increases, the profit obtained by merchant intermediaries decreases as they face more competition, and therefore the referral mode becomes more prominent in equilibrium. In fact, for any finite agency fee, agency becomes the dominant business model as the number of intermediaries in the market grows large.

This extension shows that when the seller has less bargaining power, then the level of downstream competition and the level of referral fee are both key determinants of the intermediaries' business model. In equilibrium intermediaries earn both referral fees and profits from buying and reselling. We also observe that if the market is sufficient large, and the seller can choose the level of referral fee, then the seller has an incentive to induce each intermediary to operate as a merchant.

Combining the two parts of our paper, we provide a theory that determines the conditions under which intermediaries sell through referrals, operate as merchants, or employ both business models. Our theory points out that the bargaining power of the seller in defining the trading protocol, the level of referral fee, and the level of competition across intermediaries are three key determinants of the business model that intermediaries adopt.

The existing literature on intermediation has predominately studied why intermediaries exist. The basic premise is that there are frictions that may prevent buyers and sellers from transacting, and that intermediaries are endowed with a technology that reduces these frictions. A large number of papers have focused on search frictions (Gehrig, 1993; Johri & Leach, 2002; Rubinstein & Wolinsky, 1987; Shevchenko, 2004; Smith, 2004; Shi & Siow, 2012; Watanabe, 2010, 2013; Yavas, 1992, 1994, 1996). The literature has then moved to understand the implications of intermediation for markets, by taking as exogenous the intermediaries' business model.

Other strands of the literature have studied the two business models in isolation. For example, the literature on resale (Calzolari & Pavan, 2006; Jehiel & Moldovanu, 1999; Zheng, 2002, for example) and the literature on intermediation in networks (Blume, Easley, Kleinberg, & Tardos, 2009; Condorelli, Galeotti, & Renou, 2016; Nava, 2015) assumes that intermediaries act as merchants and focuses on understanding how the possibility of resale affects the seller's incentives and the efficiency of market outcomes. In contrast, the existing literature on referrals, for example, Arbatskaya and Konishi (2012), Park (2005), and Garicano and Santos (2004) assumes that intermediaries act as agents and study how referrals improve matching between buyers and sellers.

The referral and the merchant mode of intermediation are jointly analyzed in Johnson (2013) and Johnson (2017). These papers compare the referral mode (one in which the pricing for final customers is directly set by the wholesaler) to the merchant mode (where the retailer sets the price for final consumers). The focus is on the competition among a set of spatially differentiated products. Further, Hagiu (2007) provides a comparison of the two models based on complementarity among the products of different sellers. In contrast to our work, in these papers information is complete and the choice between the agency and the merchant mode is not endogenous.

A handful of recent theoretical papers endogenize the choice to operate as a merchant or an agent. A closely related paper is Hagiu and Wright (2014) in which the intermediary decides between operating as an agent (i.e., marketplace) or as a reseller
(i.e., a merchant). Their paper focuses on a different trade-off than the one we have highlighted. The optimal choice of the intermediary depends on the relative quality of information that the intermediary and the suppliers have about sale-enhancing marketing activities. A related trade-off is analyzed in Hagiu and Wright (2015). Abhishek, Jerath, and Zhang (2015) focus on the effect on this choice of price and demand spillovers from the electronic channel to the traditional channel. Other papers that endogenize this choice include Lu (2015) and Gautier, Hu, and Watanabe (2016) who focus on different trade-offs faced by intermediaries. See Vettas (2017) for a recent comprehensive survey.

Section 2 describes the basic model and Section 3 provides the equilibrium analysis. Section 4 characterizes equilibrium when the seller cannot set a minimum reserve price. Section 5 explores the various directions to which our results are robust and, finally, Section 6 concludes. Appendix A contains the proofs. Appendix B generalizes our results of Section 3, whereas Appendix C endogenizes referral fees both in the case where the seller has the power to set them and in the case where intermediaries do.

2 | MODEL

The seller of an indivisible object is connected to \( n \geq 1 \) intermediaries, \( I = \{1, 2, \ldots, n\} \), and each intermediary has access to a set of buyers \( B_i \). For simplicity we assume that intermediaries have exclusive access to their buyers, \( B_i \cap B_j = \emptyset \), for all \( i, j \in I \) and \( i \neq j \); we also assume that all intermediaries are connected to the same number of buyers, \( |B_i| = B \) for all \( i \in I \).\(^{13}\)

The seller and the intermediaries derive zero utility from consuming the object. Each buyer \( j \) has a private consumption value \( v_j \in [0, 1] \), distributed according to the CDF \( F \), with density \( f \). We assume that all values are independently drawn and are buyers’ private information. All players are risk neutral. The seller and intermediaries maximize profit whereas buyers maximize their surplus.

We consider the following game, which unfolds in three stages:

2.1 | First stage: Business model phase

Given an exogenous referral fee, \( \kappa \geq 0 \), intermediaries choose their business model simultaneously: Each intermediary \( i \in I \) decides whether to operate as a merchant, taking action \( M \), or as an agent, taking action \( A \). If intermediary \( i \in I \) decides to operate as an agent, she refers her buyers to the seller in exchange for referral payments \( \kappa B \), and exits the game. Intermediaries who adopt the merchant mode participate to the second stage.

2.2 | Second stage: Trading phase

At the beginning of the second stage, the business model of all intermediaries becomes public. The seller sets a nonnegative reserve price, and runs a second-price auction with the chosen reserve price.\(^ {14}\) Bidders in this auction are the merchant intermediaries and the buyers referred to the seller in the first stage. All bidders simultaneously submit a bid. If there are bids above (or equal to) the reserve price, the good is awarded to the highest bidder, who pays the maximum between the second highest bid and the reserve price. The game ends if no trade takes place or if a buyer gets the good. When a merchant intermediary acquires the object the game proceeds to the resale stage.

2.3 | Third stage: Re-sale phase

After acquiring the object, a merchant intermediary sets a nonnegative reserve price and runs a second price auction.\(^ {15}\) Only his buyers, that is, those in \( B_i \), can participate in the auction; it is irrelevant what players observe of the past history. The game ends at the end of the resale phase, regardless of whether or not trade took place.

We characterize perfect Bayesian equilibria in which players play un-dominated strategies on and off-path. The latter requirement implies that all participants in a second price auction always bid their own valuation. In particular, final consumers bid their private valuation, and merchant-intermediaries bid their resale value, which is the payoff they expect to obtain in the resale phase if they acquire the good in the second-stage.

The model above abstracts from ancillary costs that may arise from setting up a referral program or, more generally, organizing the sale and aims at streamlining the basic trade-off that an intermediary faces when deciding whether to operate as a merchant or as an agent. In particular, by operating as an agent, the intermediary obtains a fixed and deterministic payment from the seller, which only depends on the referral fee and the number of buyers that are connected to the intermediary. Instead, by operating as merchant the intermediary aims at buying the object from the seller at a low price and at reselling it to his buyers at a higher
price. The buying and selling price, however, depend on a number of variables, and, in particular, on the market structure and the trading protocol—which therefore crucially affect the choice of the intermediary in equilibrium.\footnote{16}

Importantly, the way this basic trade-off resolves in equilibrium is not contingent on the many specific assumptions of this framework and we clarify this when we present the results and elaborate on them in Appendix B. We do remark here that all the results we obtain extend, qualitatively, to the case where the referral payment to an intermediary is a percentage of the seller’s profit (a “commission”), rather than a fixed fee per-buyer fee.

3 | PRICING FLEXIBILITY AND THE RISE OF THE REFERRAL MODE

Our first result shows that, as long as referral fees are positive, that is, $\kappa > 0$, in equilibrium all intermediaries adopt the referral mode.

Theorem 1. In the game where trading occurs via second price auction with reserve price, for all $\kappa > 0$, there is a unique equilibrium outcome where all intermediaries adopt the referral mode.

To understand the result, suppose there is an equilibrium where some intermediary $i$ chooses to operate as a merchant with positive probability. Then, consider that the seller chooses a reserve price knowing that $i$ is a merchant. According to the equilibrium logic, the seller also knows the expected payoff that merchant intermediary $i$ obtains, should he acquire the object and resell it to buyers $B_i$, $i$'s resale value denoted by $V_i$. The key observation is that the expected demand of the seller is inelastic for prices below $V_i$ and because there is only one good for sale, the seller never sets a reserve price lower than $V_i$. But this implies that merchant intermediary $i$ makes an expected payoff of zero in the seller’s auction. Hence, the intermediary strictly gains by adopting the referral mode in the first stage.

Appendix B investigates the robustness of Theorem 1. We show that the result extends to asymmetric buyers, to asymmetric intermediaries, for example, $|B_i| \neq |B_j|$, to the possibility that each intermediary can choose to refer any subset of their buyers,\footnote{17} to the case where intermediaries have information about the number of bidders and their values of bidders that is not available to sellers, and to different trading mechanisms.\footnote{18} Heuristically, what matters for the result of Theorem 1 is the ability of the seller to rank merchant intermediaries based on their resale values, and to have trading mechanisms that allow the seller to extract the resale value of some of these intermediaries.

We now consider equilibrium payoffs. We call integrated-industry profit, and denote it by $\Pi^*$, the revenue that the seller achieves, \textit{ex ante}, if she has direct access to all buyers. The next result follows directly from Theorem 1.

Corollary 1. In the game where trading occurs via second price auction with reserve price, in the unique equilibrium outcome when $\kappa > 0$, the sum of the \textit{ex ante} expected payoffs of seller and of the intermediaries is equal the integrated-industry profit $\Pi^*$. The expected payoff of the seller is $\Pi^* - nB\kappa$, whereas each intermediary obtains $B\kappa$.

Hence, under the considered trading protocol, the fact that all intermediaries refer their buyers induces an outcome that is equivalent to one that could be obtained through a more typical vertical agreement between the seller and the intermediaries (e.g., royalty contracts or vertical integration). For instance, under both vertical integration and an appropriately designed royalty contract, the aggregate profit of the seller and the intermediaries is the integrated industry profit $\Pi^*$. Incidentally, this begs the question of why the referral mode is so widely used, instead of, say, royalty contracts. One obvious answer is that royalty contracts become very complicated in the presence of multiple intermediaries, whereas referral schemes work independently of the number of intermediaries.\footnote{19}

That the option of referring at a positive fee and different vertical agreements may have similar effects on market outcomes is of interest to policy makers and competition authorities who are often concerned that vertical contracts may restrict competition and decrease aggregate welfare. The following result shows that, pretty much like the classic vertical agreements that alleviate inefficiencies relating to double marginalization when there is a single intermediary, referrals are likely to have positive effects on aggregate welfare.

More precisely, consider the case in which intermediaries have no choice but to operate as merchants. Because intermediaries are symmetric, the seller will set a reserve price equal to the common resale value of the intermediaries and will end up selling at that price, to one intermediary at random. The winning intermediary will then run a second price auction with an optimally chosen reserve price where only the connected buyers will participate. The following result compares welfare under the equilibrium just outlined, with welfare under the equilibrium of Theorem 1.

Corollary 2. In the game where trading occurs via second price auction with reserve price, in the unique equilibrium outcome when $\kappa > 0$, the sum of the \textit{ex ante} expected payoffs of seller, the \textit{ex ante} expected payoffs of the intermediaries, and the \textit{ex ante}
expected payoffs of all buyers is weakly larger than their joint ex ante expected payoff for the equilibrium of the game in which intermediaries have no choice but to operate as merchants.

When all intermediaries adopt the referral mode, the inefficiencies that obtain are the result of the positive reserve price that the seller sets, which implies that with positive probability trade will not occur. When all intermediaries are merchants, there are additional inefficiencies that arise from pure mis-allocation. Even when all buyers have a valuation above the reserve price, inefficiencies occur because the object can end up to intermediary $i$, but it is intermediary $j \neq i$ who has access to the highest valuation buyer. Hence, allowing for referrals leads to a more efficient outcome than in the case where referrals were banned.

### 4 | THE EMERGENCE OF MIXED AGENCY–MERCHANT MODES

We now relax the ability of the seller to react to the choice of the business model adopted by the intermediaries. In particular, we consider a game where in the trading phase and in the resale phase the owner of the object runs a second price auction without a reserve price. A situation where the highest reserve price that an upstream seller can charge is zero could naturally arise when there are multiple competing sellers. A key insight of the literature on competing auctioneers (McAfee, 1993; Peters, 1997) is that, in large economies, competition of sellers for buyers has a Bertrand-style flavor and leads to reserve prices equal to a seller’s value—zero in our case. So viewed this way, this section studies, in a somewhat reduced form, the case of competing sellers.

The fact that the seller cannot set the reserve of his choice, is capturing reduced bargaining power of the seller vis-à-vis intermediaries. In fact, as we shall see, in this case an intermediary may expect to obtain a positive profit by adopting the merchant mode. Depending on the level of referral fees, intermediaries may adopt both the referral mode and the merchant mode in equilibrium.

We proceed by determining the intermediaries’ payoffs in the trading phase. There are three cases to consider. In the first case, all intermediaries adopt the referral mode, and their individual payoff is $Bk$.

In the second one, at least two intermediaries adopt the merchant mode. Because intermediaries are symmetric, the two merchant intermediaries have the same continuation value from reselling, and therefore their bid, in the auction run by the seller, will be the same and will be equal to their resale value. This implies that the expected payoff of the merchant-intermediaries is zero in such an auction.

The third, and final, case is one where all intermediaries adopt the referral mode, with the only exception of a single intermediary, who adopts the merchant mode. In this case, the merchant intermediary bids her resale value. If we denote by $v(x;Y)$ the random variable corresponding to the $X$’th highest value out of $Y$ draws from the CDF $F$, we have that the resale value of a merchant-intermediary is $V(B) = E[v_{2:B}]$. At the beginning of the trading phase, the merchant intermediary competes in the auction with $(n - 1)B$ buyers (i.e., those referred by other intermediaries) and therefore his expected acquisition payoff, $A(n, B)$, is equal to the expected profit of a participant in a second-price auction with $(n - 1)B$ opponents whose valuations are drawn from $F$:

$$A(n, B) = F(V(B))^{(n-1)B} \left[ V(B) - E[v_{2:(n-1)B} \mid v_{2:(n-1)B} < V(B)] \right] = \int_0^{V(B)} F(x)^{(n-1)B} \, dx,$$

where the last equality follows from standard results in auction theory. It is easy to verify that $A(n, B)$ is strictly decreasing with $n$ and tends to $0$ as $n$ grows large.

The strategic situation that intermediaries face in the referral phase, thus, reduces to the following. The payoff of an intermediary who adopts the merchant mode is $0$ whenever at least one other intermediary adopts the merchant mode. It is positive and equal to $A(n, B)$ when all other intermediaries adopt the referral mode. The payoff of an intermediary who adopts the referral mode is $\kappa B$ regardless of the choice of the other intermediaries. Table 1 represents this game for the case of two intermediaries; Theorem 2 identifies equilibria in the general case.

**Theorem 2.** Consider the game where trading occurs via a second price auction without reserve price.
If $\kappa B > A(n, B)$ then there is a unique equilibrium where all intermediaries adopt the referral mode.

If $\kappa B < A(n, B)$ then (i) there is a set of pure strategy equilibria, in which there is only one intermediary who adopts the merchant mode and the other intermediaries adopt the referral mode; (ii) there is a unique mixed-strategy equilibrium (in addition to the pure ones) where each intermediary adopts the referral mode with probability $a(\kappa, n, B) = \left(\frac{\kappa B}{A(n, B)}\right)^{n\alpha}$, and the merchant mode with the remaining probability.

When referral payments are not too high, the equilibrium predicts that intermediaries may adopt heterogeneous business models. Every pure strategy equilibrium is asymmetric: one intermediary adopts the merchant mode and all the other intermediaries adopt the referral mode. There is a unique mixed strategy equilibrium and it is symmetric. In this equilibrium intermediaries are indifferent between adopting the referral mode and cashing the referral payment, and adopting the merchant mode, thus facing some inventory risk, but potentially obtaining a high profit from reselling.

We now present some comparative statics results, which are a direct corollary of our previous theorem.

**Corollary 3.** The following comparative statics obtain for the mixed strategy equilibrium in the game where trading occurs via a second price auction without reserve price:

- Suppose $\kappa B < A(n, B)$: (i) the probability that an intermediary adopts the referral mode, denoted $a(\kappa, n, B)$, is strictly increasing in $\kappa$, with $a(0, n, B) = 0$ and $a(A(n, B) / B, n, B) = 1$; (ii) $a(\kappa, n, B)$ is strictly increasing in $n$.
- There exists a finite and positive threshold $\bar{\kappa}(\alpha) > 0$ so that for all $\kappa > \bar{\kappa}(\alpha)$ there exists a unique equilibrium in which $a(\kappa, n, B) = 1$, that is, all intermediaries adopt the referral mode.

When the referral fee falls, the referral mode becomes less attractive, and therefore intermediaries adopt the merchant mode more frequently, that is, $\alpha$ declines. When referral fees become negligible, intermediaries become merchants with probability one. In contrast, when the number of intermediaries grows, a merchant intermediary faces more competition, and, ceteris paribus, her profit declines. As a consequence, intermediaries adopt the referral mode with higher probability. In fact, even if referral payments are negligible, when the number of intermediaries is sufficiently large, the equilibrium predicts that all intermediaries adopt the referral mode. This result resonates with that of Theorem 1. In Theorem 1 the referral mode prevails because the reserve price allows the seller to extract the resale value of each merchant intermediary. Here, it obtains because intermediaries’ competition erodes the profit that merchant intermediaries expect by reselling.

We conclude this section by discussing the welfare properties of these equilibria. When there are $n_A$ agent intermediaries, the bidders in the seller’s auction are $n_A B$ referred buyers and $n - n_A$ merchant intermediaries. The highest bid of the referred buyers is a random variable $v_{(1:n_A)}$, whereas merchant intermediaries bid their resale value $\mathbb{E}[v_{(2:B)}]$. To evaluate total surplus we need to consider two possibilities. With probability $\mathbb{P}\{v_{(1:n_A)} > \mathbb{E}[v_{(2:B)}]\}$ one of the referred buyers wins the auction, in which case the total expected surplus is

$$
\mathbb{E}[v_{(1:n_A)} | v_{(1:n_A)} > \mathbb{E}[v_{(2:B)}]];
$$

and the remaining probability a merchant intermediary wins the auction, in which case the surplus the intermediary generates is the expected highest valuation of the buyers accessed by the winning intermediary, that is, $\mathbb{E}[v_{(1:B)}]$. Hence, if we let $W(n_A)$ be the welfare generated when $n_A \leq n$ intermediaries adopt the referral mode and the remaining $n - n_A$ intermediaries adopt the merchant mode, we obtain that

$$
W(n_A) = \mathbb{P}\{v_{(1:n_A)} > \mathbb{E}[v_{(2:B)}]\}|\mathbb{E}[v_{(1:n_A)} | v_{(1:n_A)} > \mathbb{E}[v_{(2:B)}]] +
\mathbb{P}\{v_{(1:n_A)} < \mathbb{E}[v_{(2:B)}]\}|\mathbb{E}[v_{(1:B)}].
$$

(1)

It is easy to verify that $W(n_A)$ is strictly increasing in $n_A$ and that when all intermediaries are agents, $n_A = n$, the outcome is ex post efficient. In the asymmetric equilibrium $n_A = n - 1$ and therefore the equilibrium welfare is $W(n - 1)$. Inefficiencies occur because the object may be allocated to a referred buyer, and, yet, the merchant intermediary accesses buyers with higher valuations. In the mixed strategy equilibrium $n_A$ is a random variable that follows a binomial distribution, and so the associated expected surplus is

$$
W_{\text{mix}}(\alpha) = \mathbb{E}_{n_A}[W(n_A)] = \sum_{n_A=0}^{n} \binom{n}{n_A} (\alpha^{n_A}(1 - \alpha)^{n-n_A}) W(n_A).
$$

(2)
Because $W(\alpha)$ is strictly increasing in $\alpha$, and because an increase in $\alpha$ leads to a first-order stochastic shift in the distribution of the random variable $n_\alpha$, it follows that $W_{\text{mix}}(\alpha)$ is strictly increasing in $\alpha$.

Combining these observations with the comparative statics results in Corollary 3 we obtain the following result:

**Proposition 1.** The following hold in the game where trading occurs via a second price auction without reserve price.

1. There exists $\kappa \in (0, A(n, B)/B)$ such that $W(n - 1) > W_{\text{mix}}(\alpha(k, n, B))$, that is, the asymmetric equilibrium generates a higher welfare than the symmetric mixed equilibrium, if and only if $\kappa < \kappa$.
2. There exists a finite and positive $\tilde{n}$ such that for all $n > \tilde{n}$ the mixed equilibrium outcome is ex post efficient.
3. As long as $\kappa > 0$, regardless of the equilibrium play, the associated welfare is higher than the welfare generated when referrals are not allowed and therefore all intermediaries are merchants.

Part 1 shows that when $\kappa$ is sufficiently low, in the mixed equilibrium the merchant mode is prevalent and so the welfare is higher in the asymmetric equilibrium. Part 2, an immediate consequence of Corollary 3, shows that when there is enough competition among intermediaries, in equilibrium intermediaries adopt the referral mode with probability one and therefore the outcome is ex post efficient. The last part of the proposition shows that referrals induce equilibrium outcomes, which are more efficient relative to a situation in which intermediaries are forced to be merchants.

## 5 | MAIN LESSONS AND ROBUSTNESS

### 5.1 | Alternative trading protocols

Our results highlight the importance of the protocol regulating trade between the seller and the intermediaries in determining the business model chosen by latter. We looked at two somewhat extreme cases. First, we considered the case in which the seller has full bargaining power against the intermediaries. Second, we considered the case in which the deck is stacked against the seller and trade takes place by means of an efficient auction. In the first case, all intermediaries adopt the referral mode; in the second case, both referral and merchant mode may coexist. As we pointed out, the key determinant of specialization in referral is that the seller is able to flexibly price out of the market merchant intermediaries. These results beg the question of what would be the effect of alternative trading protocols. Our analysis suggests that a specialization in referral will arise as long as the seller has some degree of pricing flexibility toward intermediaries and can charge intermediary-specific prices. On the other hand, a mixed model will arise if the seller has limited bargaining power in the selling stage and, as a consequence, less pricing flexibility.

Consider, for instance, the case in which the reserve price is set by the seller before merchants make their business model choice. In this case the seller can induce full referral by committing to a high reserve price in the sale phase. The seller still retains enough flexibility and the outcome is one with full referral, even if the reserve price is chosen before the merchants take their business mode decision. On the other hand, consider the case of a second price auction where the reserve price is exogenous and somewhere between zero and the optimal reserve price. In this case the seller has no flexibility and a hybrid market structure, with merchants and agents, is likely to arise. In fact, the analysis we performed for the case of no reserve price extends to this case when we take into account that the single-merchant net profit $A(n, B)$ will be now lower as a result of the higher reserve price.

### 5.2 | Vertical contracts: Nonlinear tariffs, royalties, profit-sharing

In many sectors, vertical contracts between upstream and downstream firms can be quite complex. In this paper we have looked at two cases. In the first case, the seller allows the intermediary complete freedom in the reselling process he uses—so the intermediary chooses the profit-maximizing one given his buyers namely a second-price auction with a reserve price. Given that in our set-up this procedure is the profit-maximizing one, there is no benefit from using nonlinear tariffs. In the second case, the intermediary, much like the seller, cannot set a positive reserve price. In this sense the resale stage is contractible and the contract restricts the choice of the intermediary. Interestingly, because the upstream seller faces the same restriction, this turns out that it could be beneficial for the intermediary.

Throughout the paper, we assume a simple linear referral fee. The main unraveling force behind Theorem 1 extends if this fee is accompanied by a commission or a royalty, which can depend arbitrarily on the intermediaries' profits from resale. The
same holds for the trade-offs that determine the choice between the agent versus merchant mode when the selling protocol is a second price auction without reserve.

5.3 | Superiorly informed intermediaries

As we explore in Appendix B, the main insight of Theorem 1 remains true if intermediaries have superior information about the number of buyers they have or their values. Also, if intermediaries have superior information about their buyers' willingness to pay vis-à-vis the seller, the seller can extract this information after referrals for a negligible commission fee. Observe that in this case a further inefficiency akin to double marginalization is present when intermediaries operate as merchants. The seller exercises market power by restricting supply against intermediaries who, in turn, exercise it against buyers.

5.4 | Endogenous referral fees

We have seen that referral fees or commissions are simple and robust tools that help eliminate inefficiencies. How the resulting higher welfare is split among parties depends on their relative bargaining powers. It is clear from the statement of Theorem 1 that if the seller has pricing flexibility and full control of the referral fee, he will set it arbitrarily close to zero. It is less obvious what happens when intermediaries set these fees. We have analyzed this scenario in Appendix B and have established that, perhaps surprisingly, there is a continuum of equilibria where referral fees range from one where the seller extracts $\Pi^\ast$, to the one where intermediaries extract the entire industry profit.

When the seller lacks pricing flexibility, the level of the referral fee determines whether all intermediaries operate as merchants, agents, or if a mixed mode will prevail. As in the previous case, it can be shown that if the intermediaries choose the referral fee, there is an equilibrium where the referral model is adopted with probability one and intermediaries collectively extract the entire surplus. In contrast with the previous case, when the seller cannot condition his pricing strategy on the market structure, there is a minimum referral fee that the seller must set in order to induce referral by all intermediaries. Although we have not been able to prove that the seller will always want to induce full referral, it can be shown that if the number of buyers that each intermediary accesses is large enough, the seller will strictly prefer to choose the minimum fee that guarantees full referral.22

5.5 | Different market segments, demographics, etc

In the context of our model, agency's welfare superiority is not limited to environments in which the seller is ultimately connected to buyers with the same expected demand. Coming to an environment with asymmetric buyers, referrals increase efficiency if there are a number of different consumer types (i.e., different distributions of valuations) and all the intermediaries are connected to sets of buyers that contain all the different consumer types. Under this scenario, the reserve price that the seller or intermediaries charge is the same. Therefore in the trading phase, the auction of the seller, in which all buyers participate, cannot introduce additional distortions compared to those that each intermediary would introduce in the resale phase anyway (Theorem B.3 in Appendix B).

More generally the referral mode is associated with higher welfare for the aforementioned reasons, whenever intermediaries find it optimal to set a reserve price weakly higher than the seller. The same is true for environments where the selling protocol to consumers is determined by factors beyond the control of the seller and of the intermediary—for example, the retail price is exogenously fixed due to regulatory requirements or there is intense competition in the downstream market.

6 | CONCLUSION

The literature on intermediation has primarily focused on identifying channels that make intermediation valuable and on explaining the presence of intermediaries in modern marketplaces. As such, the modus operandi of the intermediary was exogenous in the analysis.

Broadly, there are two business models that are widely adopted by intermediaries, the referral mode and the merchant mode. As we have discussed in length, under the referral mode, intermediaries receive a fee—a commission—for the introduction of a client to the seller. Under the merchant mode, intermediaries buy and resell.

In this paper we present a theory that makes the choice of the intermediary's business model endogenous. We highlight one of the trade-offs that intermediaries are exposed to in determining whether to operate as agents or merchants. We consider a
number of key variables but, most important, we analyze how the equilibrium industry structure is determined by the negotiation and trading protocols that characterize the industry.

We see that when the seller can condition her minimum price on whether or not intermediaries are among the buyers, the referral mode prevails and it is the one that is associated with the highest allocative efficiency. This insight remains true in richer environments. When the initial seller lacks such price flexibility, both the agency and the merchant mode can coexist.

We also provide a welfare analysis of the referral mode, which emphasizes the ability of this business model to solve the allocative inefficiencies that arise with the standard merchant mode. In our model these inefficiencies result from incomplete information. They relate to the well-known problem of double marginalization in the merchant mode and to the risk that the object is not allocated to the highest value client because the wrong intermediary acquires it.

Our analysis points out that first, the referral mode can be welfare enhancing and, second, higher final prices resulting from the referral mode need not be an expression of market power, but rather a result of a more efficient allocation of the goods. These observations may provide useful insights for competition policy in intermediated markets.

We make a number of simplifying assumptions for tractability. As discussed in Section 5, our results are robust to a number of nontrivial perturbations of the environment. Still, a number of questions remain open and are left for future work. For instance, our analysis relies on the good being scarce, perhaps due to a high marginal cost of production. This might not be the right assumption in many cases of interest. Also, although in Appendix B we show that full specialization of all intermediaries in the referral mode arises robustly under many trading protocols where the seller has pricing flexibility, the absence of flexibility indeed generates different outcomes depending on the trading protocol. For example, it would be interesting to consider the case of posted prices in both the upstream and downstream market. Finally, in our model the referral fee is treated as exogenous and making it the choice of the seller or of the intermediary leads to some stark results (see Appendix C). However, in general, both the business model and the referral fee affect both sellers and intermediaries. Therefore, it is not obvious that these two variables are exogenous or simply a choice left to one of the two parties.

**APPENDIX A: PROOFS**

*Proof Theorem 1.* Let \( r^* \), such that \( r^* - \frac{1-F(r^*)}{f(r^*)} = 0 \). We first show that in equilibrium all intermediaries adopt the referral mode with probability one. When an intermediary chooses to be a merchant he acquires the object from the seller and resells it to his \( B \) buyers via second price auction with the revenue-maximizing reserve price \( r^* \). Denote by \( V \) the expected profit of the intermediary in this resale auction with \( B \) participants who face a reserve price of \( r^* \). This is how much is worth to the intermediary to obtain the object, that is the intermediary’s valuation. Notice two things: First, the seller can fully anticipate \( V \). Second, whenever an intermediary chooses to be a merchant, that intermediary must bid \( V \) in the auction that the seller runs. Then, in equilibrium, the seller’s optimal reserve price depends on whether or not there exist at least one intermediary that chose to be a merchant: In particular, the seller posts a reserve of \( r^* \) when all intermediaries are agents, a reserve of \( \bar{r} = V \) when all intermediaries are merchants and a reserve of \( \bar{r} = \max\{r^*, V\} \) otherwise. For a contradiction, suppose there is an equilibrium where intermediary \( i \) is a merchant with strictly positive probability. When intermediary \( i \) adopts the merchant mode she obtains an expected payoff of 0 because she bids \( V \) and the reserve price is \( \bar{r} \geq V \). Hence, intermediary \( i \) strictly prefers to adopt the referral mode, which gives a payoff of \( \kappa B > 0 \). It is easy to show that the following strategy profile is an equilibrium: each intermediary adopts the referral mode, reserve prices are set as described above, and bidding is truthful.

*Proof of Theorem 2.* Recall that if an intermediary chooses to be a merchant she expects to obtain a payoff of \( A(n, B) \) whenever all other intermediaries are agents, and a payoff of 0 when at least another intermediary is a merchant, and that \( A(n, B) \geq 0 \). We first characterize the set of equilibria in which intermediaries play a pure strategy. Suppose two or more intermediaries choose the merchant mode with probability one and the remaining intermediaries adopt the referral mode. Then, the merchant intermediaries obtain a payoff of 0. Deviating and becoming an agent pays \( \kappa B > 0 \). Suppose all intermediaries adopt the referral mode, so that each gets \( \kappa B \). If intermediary \( i \) deviates and adopts the merchant mode, she will get \( A(n, B) \). Hence an equilibrium where all intermediaries adopt the referral mode exists if and only if \( \kappa B \geq A(n, B) \). The last possibility is that intermediary \( i \) adopts the merchant mode and the other intermediaries are agents. Note that an agent intermediary strictly loses if she switches to the merchant mode. The merchant intermediary obtains \( A(n, B) \) and if she becomes an agent she gets \( \kappa B \), and so she prefers to be a merchant intermediary whenever \( A(n, B) \geq \kappa B \).

We conclude by considering equilibria in which intermediaries mix between adopting the agency and the merchant mode. It is immediate to see that if a mixed strategy equilibrium exists, it has to be symmetric. So, suppose each intermediary becomes an agent with probability \( a \in (0, 1) \). Then the payoff to an intermediary who becomes an agent is \( \kappa B \), and the payoff to an intermediary who becomes a merchant is \( a^{n-1} A(n, B) \). In equilibrium \( \kappa B = a^{n-1} A(n, B) \), and the result follows.
Proof of Proposition 1. To see point 1, recall the definition of $W_{mix}$ given in (2). If $\alpha(k, n, B) = 0$ then $\Pr(n_A = 0) = 1$ and $W_{mix} = W(0)$; whereas when $\alpha(k, n, B) = 1$ then $\Pr(n_A = n) = 1$ and $W_{mix} = W(n)$. Because $W(n_A)$ is strictly increasing in $n_A$ and $W_{mix}$ is continuous in $\alpha$, the intermediate value theorem implies that there exists $\alpha^* \in (0, 1)$ such that $W_{mix} > W(n - 1)$ for $\alpha(k, n, B) > \alpha^*$ and the converse holds otherwise. The proof of this point is concluded by observing that $\alpha(k, n, B)$ is continuous in $k$ and $\alpha(0, n, B) = 1$ whereas $\alpha(A(n, B), B, n, B) = 1$. Therefore, again by the intermediate value theorem, the value $k$ must exist in the interval $(0, A(n, B)/B)$.

Point 2 follows from Corollary 3 and from the fact that when all intermediaries operate as agents, the outcome is ex post efficient. To see point 3 note that welfare in the pure merchant mode is $W(0)$. Then, observe that $W(n_A)$ is strictly increasing in $n_A$. It follows that $W(n) > W(n - 1) > W(0)$. Because $\Pr(n_A \neq 0) > 0$ when $\kappa > 0$, we also conclude that $W_{mix}(\alpha(k, n, B)) > W(0)$ for $\kappa > 0$.

APPENDIX B: ROBUSTNESS OF RESULTS

In this appendix, we introduce a generalization of the model presented in Section 2. Intermediaries can be connected to arbitrarily different sets of buyers and to be privately informed about that set. The seller can choose in the trading phase any selling procedure. More precisely, the seller of an indivisible object is connected to a set of intermediaries denoted by $I$ and each intermediary $i \in I$ is linked to a set of buyers $B_i$. Intermediary $i$ privately observes the set $B_i$ whose members are drawn from a finite set of potential buyers $B$. Let $\hat{B}_i$ denote the expected number $i$'s buyers. With a slight abuse of notation we denote by $I$ also the number of intermediaries.

We maintain the assumptions that intermediaries have exclusive access to their buyers, that is, $B_i \cap B_j = \emptyset$ for all $i \neq j$; that the assignment of buyers is independent across intermediaries; that the seller and the intermediaries have zero consumption value, whereas each consumer has a private consumption value for the object, $v_i \in [\bar{v}_i, \bar{v}_i]$, with $\bar{v}_i > 0$. Valuations are distributed independently of one another and of the assignment of buyers to intermediaries according to a joint distribution. The seller and intermediaries are interested in maximizing their profits and the buyers their surplus. All parties are risk neutral.

Intermediaries have superior information about their buyers' valuations compared to the seller. Formally, intermediary $i$ observes a signal $s_i \in S_i$—where $S_i$ is finite and stands for the set of possible signals receivable by intermediary $i$. This specification encompasses, among others, the case where intermediaries have access to the same information and the case where they only obtain information about their own buyers. A type of intermediary $i$ is $t_i \equiv (s_i, B_i)$, and $\mathcal{T}_i$ is the set of his possible types. Everything that it is not privately observed—including the structure of intermediaries' information—is common knowledge.

We look for perfect Bayesian equilibria of the following game:

B.1 Referral phase

Given referral fees $\{\kappa_i\}_{i \in I}$, each intermediary conditional on his type $t_i = (s_i, B_i)$ decides which subset of buyers to refer, $\hat{B}_i \subseteq B_i$. The referral fee $\kappa_i$ determines the payment to $i \in I$ per referred buyer.

B.2 Trading phase

**Stage 1 (Sale):** The seller observes who is referred and then selects a procedure, formally a mechanism, to sell the object to intermediaries and referred buyers. Buyers and intermediaries can opt out and receive their outside option, which is zero. The seller commits to the outcome of the mechanism and by the revelation principle it is without loss to assume that he selects among direct incentive-compatible and individually rational mechanisms. The game ends either if no trade takes place or if a buyer obtains the good. If intermediary $i \in I$ obtains the object, the game proceeds to the following stage. Who participates to the mechanism is common knowledge.

**Stage 2 (Resale):** If $i \in I$ acquires the object, he selects a feasible direct mechanism to resell the object to the buyers that he has not referred. The game ends at the end of the resale phase.

The following Theorem B.1 shows that in this more general environment all intermediaries refer all their buyers extending Theorem 1. Theorem B.2 extends the result of Corollary 2 and so confirms the robustness of the insight that referrals lead to outcomes that are equivalent to the outcomes obtained by vertical contracting. The discussion on welfare formalized in Theorem B.3 extends to environments in which referrals are beneficial in terms of welfare.

**Theorem B.1 (Unraveling).** Suppose that $\kappa_i > 0$ for all $i \in I$. In every equilibrium, all intermediaries refer all their buyers to the seller regardless of their types.
Proof. We prove the result in two steps: Step 1 proves Lemma B.1; Step 2 uses Lemma B.1 to conclude the proof. At the beginning of the trading stage, the seller chooses an incentive-compatible direct mechanism that satisfies participation constraints (i.e., provides payoff higher than or equal to zero to all participants). Participants are intermediaries and buyers. Let $t_i$ denote a generic profile of types of all the participants in the mechanism. We write $t_i \equiv t_i$ when agent $i$ is a buyer. When agent $i$ is an intermediary, $t_i \equiv (s_i, B_i) \in T_i(B_i)$, where $T_i(B_i) \subseteq T_i$ denotes the support of the seller's posterior following observed referrals $B_i$. When intermediary $i$ has referred $B_i$, he has obtained the object, we denote by $V_i(t, B_i)$ his expected revenue from resale when the profile of types is $t$—this is intermediary $i$'s resale value.

B.3 Step 1: Zero surplus from resale

Let $T_i^*(B_i) \equiv \{(s_i, B_i) \in T_i(B_i) \mid B_i \subsetneq B_i\}$ be the set of intermediary $i$'s types that, with some probability, have not referred all their buyers and therefore have strictly positive resale value. Given reports $(t_i, l_{-i})$, let $p_i(t_i, l_{-i})$ be the probability that $i$ gets the good and let $x_i(t_i, l_{-i})$ be the expected payment. Then, under truth telling, the expected continuation payoff of an intermediary $i$ of type $t_i = (s_i, B_i)$ who has referred $B_i$ is

$$U_i(t_i) = \mathbb{E}_{l_{-i}} \left[ p_i(t_i, l_{-i})V_i(t_i, l_{-i}, B_i) - x_i(t_i, l_{-i}) \right] \geq 0,$$

(B.1)

where the inequality follows from the fact that we impose voluntary participation constraints.

Lemma B.1. Consider an equilibrium history of the game where intermediary $i$ has referred a set of buyers $B_i \subsetneq B_i$. Then, there exists a type $t_i \in T_i^*(B_i)$ such that $U_i(t_i) = 0$.

Proof. Take a mechanism $(p, x)$ that is feasible and maximizes the seller's revenue. For a contradiction, assume that $U_i(t_i) > 0$, $\forall t_i \in T_i^*(B_i)$.\footnote{We use $\forall$ to denote the quantifier 'for all'} Let $u = \min_{t_i \in T_i^*(B_i)} U_i(t_i)$. We partition the set of types that have referred all their buyers, that is, $T_i(B_i) \setminus T_i^*(B_i)$, into two sets: $Z_i \equiv \{t_i \in T_i(B_i) \setminus T_i^*(B_i) \mid U_i(t_i) \geq u\}$ and $H_i \equiv T_i(B_i) \setminus \{T_i^*(B_i) \cup Z_i\}$.

Consider the following alternative mechanism $(\hat{p}, \hat{x})$, whereby $\hat{p}_i = p_i$ and $\hat{x}_j = x_j$ for all $j \neq i$ and:

$$\hat{p}_i(t_i, l_{-i}) = \begin{cases} p_i(t_i, l_{-i}) & \text{if } t_i \in T_i^*(B_i) \cup Z_i, \\ 0 & \text{otherwise,} \end{cases}$$

$$\hat{x}_i(t_i, l_{-i}) = \begin{cases} x_i(t_i, l_{-i}) + u & \text{if } t_i \in T_i^*(B_i) \cup Z_i, \\ 0 & \text{otherwise.} \end{cases}$$

We denote by $U_i(t'_i | t_i)$ and $\hat{U}_i(t'_i | t_i)$ the expected payoff of $t_i$ from reporting $t'_i$ in mechanisms $(p, x)$ and $(\hat{p}, \hat{x})$, respectively.

First, we show that $(\hat{p}, \hat{x})$ is feasible. It satisfies participation constraints trivially. Incentive compatibility follows because, by assumption, $(p, x)$ is incentive compatible and, by construction, for all $t_i \in T_i(B_i)$ we have: (1) $\hat{U}_i(t'_i | t_i) = U_i(t'_i | t_i) - u \geq 0$ for all $t'_i \in T_i^*(B_i) \cup Z_i$, and (2) $\hat{U}_i(t'_i | t_i) = 0$ for all $t'_i \in H_i$.

Second, we show that $(\hat{p}, \hat{x})$ generates strictly higher expected revenue for the seller than $(p, x)$. All $t_i \in T_i^*(B_i) \cup Z_i$ pay strictly more under $(\hat{p}, \hat{x})$ than under $(p, x)$. Because $t_i \in H_i$ have zero resale value and $(p, x)$ is feasible, then the expected payment from $t_i$'s perspective is at most zero, that is, $E_{l_{-i}}[x_i(t_i, l_{-i}) | t_i] \leq 0$. Hence,

$$\mathbb{E}_{t_i \in H_i} \left[ E_{l_{-i}}[x_i(t_i, l_{-i}) | t_i] \right] \leq 0 = \mathbb{E}_{t_i \in H_i} \left[ E_{l_{-i}}[\hat{x}_i(t_i, l_{-i}) | t_i] \right],$$

where the equality follows because, by construction, all $t_i \in H_i$ always pay zero under $(\hat{p}, \hat{x})$.

B.4 Step 2: Unraveling

We conclude the proof by contradiction. Let $k_i > 0$ and suppose that $T_i^*(B_i)$ is nonempty. Lemma 1 implies that there exists $t_i \in T_i^*(B_i)$ such that $U_i(t_i) = 0$. But then type $t_i$ will strictly prefer to refer buyers $B_i \setminus B_i$.

Theorem B.1 shows that, in equilibrium, intermediaries refer all their buyers. Because intermediaries are pooling in their referral strategy, the set of referred buyers does not convey any information to the seller about their valuations. Therefore, after the referral phase, intermediaries still possess private information that is valuable to the seller. Can intermediaries profit from this information?

The answer is no. Once the seller is connected to all buyers, the intermediaries’ information becomes payoff irrelevant, and the seller can acquire it at no cost. For instance, the seller can offer a very small fraction of his final revenue to intermediaries in exchange for their information, while committing to use the information to optimize his sale to the buyers. Then, the seller’s and
the intermediaries' interests are aligned, and intermediaries have an incentive to report their information truthfully. The use of such commission fees is widespread in industries in which the price between buyers and seller is negotiable, and intermediaries have relevant information about buyers' preferences (e.g., online markets, real estate, recruiting agencies).

This discussion, together with the unraveling result, suggest that, for a commission fee \( \alpha \) and referral fees \( \{\kappa_i\}_{i \in I} \), the seller can always obtain

\[
(1 - |I|\alpha)\Pi^* - \sum_i \kappa_i \bar{B}_i,
\]

where recall that the integrated-industry profit \( \Pi^* \), is the seller's ex ante expected revenue when he has access to all intermediaries' information and to their buyers and \( \bar{B}_i \) denotes the expected number of intermediary \( i \)'s buyers. Hence, given that the seller is free to pick \( \alpha \) arbitrarily small, we obtain the following result:

**Theorem B.2.** Suppose that \( \kappa_i > 0 \) for all \( i \in I \). In every equilibrium, the ex ante expected joint payoff of seller and the intermediaries is the integrated-industry profit \( \Pi^* \). The ex ante expected equilibrium payoff of the seller is \( \Pi^* = \sum_i \kappa_i \bar{B}_i \) whereas intermediary \( i \) obtains \( \kappa_i \bar{B}_i \).

Theorem B.2 shows that referrals allow the seller and intermediaries to maximize the sum of their expected profits, but it is silent with regard to whether or not referrals increase the efficiency of the assignment of the object to consumers. In our model, neither the seller nor the intermediaries have positive consumption value, so efficiency is measured in terms of the valuation of the buyer that acquires the good. We compare the (ex ante expected) efficiency of the equilibrium outcome of our game when referrals are and when they are not possible.

**Proposition B.1.** Referrals increase efficiency when there is one intermediary.

Proposition B.1 is an immediate implication of Theorems B.1 and B.2, and therefore we omit the proof. When referrals are not possible, the asymmetric information between the seller and the intermediary implies that, with some probability, the object does not reach the intermediary, thereby creating a potential loss in efficiency. This loss is not present with referrals. Moreover, referrals do not introduce any other inefficiencies: when there is one intermediary, the seller employs in the sale phase the same mechanism as the intermediary would employ in the resale phase.

When there are multiple intermediaries a new complication arises: when the seller aggregates a set of asymmetric buyers coming from different intermediaries, then, at the revenue-maximizing mechanism he will distort the outcome away from the efficient allocation. This happens because the optimal auction handicaps buyers whose valuations are more likely to be higher in order to intensify competition and to increase revenue. We conclude this section by describing a set of natural environments where, despite this complication, referrals increase efficiency.

**Definition B.1 (Same asymmetry).** Let \( F(B|s) \) denote the set of different distributions of valuations of the set of buyers in \( B \) conditional on information \( s \). An environment satisfies same asymmetry if, for all \( i, j \in I \), \( F(B_i|s_i) = F(B_j|s_j) = F(\cup_{l \in E} B_l|\cup_{l \in E} s_l) \), for all \( s_k \in S_k \) and \( B_k \in B_k \), \( k = i, j \).

In other words, Same Asymmetry means that there are a number of different consumer types (i.e., different distributions of valuations), but all the intermediaries are connected to sets of buyers that contain all the different consumer types. Same Asymmetry describes both environments with symmetric and asymmetric buyers. If \( F(B|s) \) is a singleton and Same Asymmetry holds, then the environment is one with symmetric buyers. An example is when all buyers are ex ante symmetric and the intermediaries' private information is about the number of buyers each is connected to. This captures industries in which the main asset of intermediaries is the large set of potential customers. Another example is when all consumers' valuations are drawn from the same distribution (i.e., the correct demand function) known to the intermediaries but not to the seller. This is descriptive of the online travel industry, in which large intermediaries like Expedia and Bookings.com have more precise information on consumers' trends than local hoteliers.

For an example of an asymmetric environment satisfying Same Asymmetry, think, for instance, social networking web sites offering targeted advertising. Each social networking web site is connected to various categories of users: male youths, whose valuation is drawn from some distribution \( f \); female youth, whose valuation is drawn from, say, \( g \); and others, whose valuation is drawn from some other distribution.

**Theorem B.3.** If Same Asymmetry holds then referrals increase efficiency.

When Same Asymmetry is satisfied, all intermediaries, conditional on their information, have access to the same set of buyers' distributions of valuations. Under this scenario, the optimal mechanism that the seller uses in the trading phase—in which all
APPENDIX C: ENDOGENOUS REFERRAL FEE

It is natural to think that the level of the referral fee $\kappa_i$ is negotiated between the seller and the intermediary $i$. We consider two polar bargaining scenarios, one in which the seller proposes the referral fees, and another in which each intermediary proposes.

C.1 Bargaining power to the seller

Suppose that, prior to the referral phase, the seller publicly proposes a referral fee $\kappa_i$, for all $i \in I$. After the announcement, the game proceeds as described in Section 2. Theorem B.2 implies that if the seller announces $\kappa_i > 0$ for all $i \in I$, he obtains

$$\Pi^* = \sum_i \kappa_i E[|B_i|].$$

Because the seller can choose $\kappa_i$ arbitrarily small, we conclude that in all equilibria of this game, he obtains $\Pi^*$. When the seller organizes referrals, the intermediary is left with no rent. In this case, if the intermediary has the ability to commit to not referring his buyers, he will choose to do so and expect a positive information rent by buying and reselling.

C.2 Bargaining power to intermediaries

We now consider the scenario in which intermediaries propose referral fees to the seller. The timing of the game is as follows. First, intermediaries simultaneously announce per-buyer referral fees to the seller. Then, the seller decides which referral fees to accept and which to reject. Intermediaries and buyers may or may not observe the fees posted by other intermediaries (and their acceptance), but who observes what is common knowledge and an intermediary observes more than his buyers. If the seller rejects all referral fees, the game moves directly to the trading phase, as described in Section 2. If at least one referral fee is accepted, the game moves to the referral phase, and intermediaries whose referral fee is accepted can refer their buyers. After referrals take place, the game moves to the trading phase, as described in Section 2.

Let $\{\kappa_i\}_{i \in I}$ be the profile of proposed referral fees. We first observe that, in every equilibrium where the seller accepts $\kappa_i > 0$, intermediary $i$ refers all his buyers to the seller (Theorem B.1) and the seller extracts all of his information at no cost (Theorem B.2).\footnote{We now show that there is a class of efficient equilibria in which all buyers are referred, the joint profit of the seller is the integrated-industry profit, and intermediaries can extract part of this profit. Every equilibrium in this class is sustained by the seller’s belief that any deviation to proposing a higher fee comes from the type of intermediary with the highest resale value. These out-of-equilibrium beliefs are natural, as the intermediaries with the highest resale value are the ones who have the strongest incentive to deviate by proposing higher fees. For these reasons, this class of equilibria survives standard equilibrium refinements, such as the intuitive criterion.\footnote{For any subset of intermediaries $\hat{I} \subseteq I$, let $\Pi^*_{\hat{I}}$ denote the payoff that the seller anticipates, when he expects to be connected to all buyers of intermediaries in $I \setminus \{\hat{I}\}$ and to obtain their information, but he neither is connected to nor has access to the information of intermediaries’ buyers in $\hat{I}$. In words, $\Pi^* - \Pi^*_{\hat{I}}$ is the marginal value to the seller from being connected to the buyers of intermediaries in $\hat{I}$ and having access to all their information.

\textbf{Theorem C.1.} \textbf{Seller proposes.} When the seller announces referral fees, there is a unique equilibrium outcome where intermediaries refer all their buyers and earn zero profits, and the ex ante expected equilibrium payoff of the seller is the integrated-industry profit $\Pi^*$.

\textbf{Intermediaries propose.} When intermediaries propose referral fees, for any profile of referral fees $\kappa^* = \{\kappa^*_i\}_{i \in I}$ such that

$$\Pi^* - \Pi^*_\hat{I} \geq \sum_{i \in I} \kappa^*_i E[|B_i|], \quad \text{for all } I \subseteq I,$$

(C.1)
there is an equilibrium in which each type of intermediary $i \in I$ proposes $\kappa_i^*$, the seller accepts the proposal and the intermediaries refer all their buyers. In this equilibrium, the ex ante expected equilibrium payoff of the seller is $\Pi^* = \sum_{i \in I} \kappa_i^* E[|B_i|]$, and the ex ante expected equilibrium payoff of intermediary $i$ is $\kappa_i^* E[|B_i|]$.

Proof. Seller proposes. Because Theorems B.1 and B.2 apply when $\kappa_i > 0$ for all $i \in I$, we know that the seller can always guarantee himself a payoff arbitrarily close to $\Pi^*$ by setting $\kappa_i > 0$ for all $i \in I$ arbitrarily small. Therefore, there is no equilibrium where the seller obtains a payoff strictly less than $\Pi^*$. To complete the proof, we need to show that there is an equilibrium in which the seller sets $\kappa_i = 0$ for all $i \in I$ and all intermediaries refer all buyers. This follows from arguments identical to the ones used to establish Theorem B.2 and are, therefore, omitted.

Intermediaries propose. For any $\kappa_i^*$, $i \in I$ that satisfies the conditions in the statement of the second part of Theorem C.1, consider the following strategy profile: For each $i \in I$, each intermediary type $t_i \in T_i$ proposes $\kappa_i^*$. The seller accepts $\kappa_i \leq \kappa_i^*$, and rejects $\kappa_i > \kappa_i^*$, for all $i \in I$. When the seller observes a proposal $\kappa_i \leq \kappa_i^*$, his beliefs about intermediary $i$’s type remains equal to his prior. When the seller observes $\kappa_i > \kappa_i^*$, he believes that intermediary $i$ is of the highest resale value type. When the seller accepts a referral fee $\kappa_i \geq 0$, the intermediary $i$ refers all his buyers. In the trading phase, the seller sets a feasible revenue-maximizing mechanism. We show that this profile of strategies is an equilibrium.

If the intermediary $i \in T_i$ proposes $\kappa_i^*$, he obtains $|B_i|\kappa_i^*$. By proposing $\kappa_i < \kappa_i^*$, he obtains $|B_i|\kappa_i < |B_i|\kappa_i^*$. If he proposes $\kappa_i > \kappa_i^*$, the seller rejects the proposal and believes that the intermediary has the highest resale value. Hence, the intermediary obtains a payoff of zero regardless of whether or not he gets the object at the trading phase.

Next, consider the seller. It is clear that it is optimal to reject $\kappa_i > \kappa_i^*$ given that her posterior belief in that case is that the intermediary is of the highest resale value type. The condition in equation (C.1) guarantees that the seller is not willing to deviate by rejecting any set of proposals of referral fees at $[\kappa_i^*]_{i \in I}$.

The findings of Theorem C.1 may help rationalize the observed heterogeneity in referral fees within and across industries. For example, in the online travel agency industry the typical commissions vary from 0% for airline tickets and car rentals to 20%–25% for hotel rooms and vacation packages.27

ENDNOTES

1 Intermediation is pervasive: according to Spulber (1996)’s calculations, it represented about 25% of US GDP in 1995. This is arguably a conservative estimate of the size of intermediation today. Since 1995, the diffusion of the Internet has boosted the creation of hundreds of new online marketplaces, where intermediaries manage the interaction between buyers and sellers (e.g., Amazon, Airbnb, Alibaba, Uber).

2 For more details, see Insider (2012) and priceline.com, Inc. (2011)

3 Expedia, Inc. (2013, p. 10)

4 Orbitz Worldwide Inc. (2012, p. 35). Orbitz also owns online travel agents such as CheapTickets and eBakers.

5 We take the existence of intermediaries as given. The literature has provided various rationales for the existence of middlemen. The main idea in the economics of intermediation is that the matching between sellers and buyers is not frictionless; intermediaries, then, emerge because they own technologies that allow to ameliorate these frictions with some comparative advantage (e.g., see Rubinstein and Wolinsky (1987)).

6 In our model, consumers cannot approach the seller directly. Nonetheless, in some cases, such as for the case of hotel rooms, the identity of the seller must be revealed prior to the sale. Our assumption still makes sense when the intermediary is a merchant, as in that case the seller has no longer the good for sale. However, in practice, it is conceivable that a consumer may attempt to bypass the agent intermediary and contact the seller directly, in an attempt to attain a rebate equivalent to the referral fee. It appears that this concern is either not substantial (e.g., in the case of Airbnb where the details of the seller are not revealed before the booking) or otherwise handled by mean or resale-price maintenance clauses.

7 The inefficiencies that are eliminated by the referral mode cannot be easily suppressed through a more traditional simple vertical agreement, such as a royalty contract, unless there is exactly one intermediary. We elaborate on this later.

8 See Rey (2003) for an extensive survey of the academic literature on vertical restraints and an account of the legal issues related to vertical restraints in the United States and the European Union.

9 The DOJ won the case against Apple and reached a settlement which forced Apple to operate as a merchant. Apple is appealing.

10 In particular, the court was also concerned that the referral mode could favor horizontal collusion among publishers, a point which was at the core of the DOJ’s case. Moreover, the source of inefficiency emphasized in our paper arises out of rationing, which is arguably not a major problem in the case of e-books.

11 Pure strategy equilibria exists and are asymmetric: one intermediary becomes a merchant and all the others adopt the referral mode.

12 Montgomery (1991) and Galenianos (2013) study the effects of allowing firms to hire through referrals for labor-market outcomes. In these two papers, hiring through a referral means hiring a social contact of a current employee. We are interested in the role of referrals in markets where trade can be intermediated and so our model of referral and the questions we pose are different from these works.

13 These assumptions are relaxed in Appendix B.
The second price auction is a natural choice in this environment. It is an optimal auction when the reserve price is appropriately chosen. With no reserve price, a second price auction essentially generates the minimum price compatible with an efficient allocation. As we illustrate in Appendix B, the fact that trading takes place through a second price auction is not crucial for our qualitative results.

As noted before, this is an optimal mechanism from the reseller's perspective.

A merchant also exposes himself to an inventory risk, that is, the risk of buying and not being able to resell. In our model, we assume that intermediaries are risk neutral. Therefore, we shut down the payoff penalty that bearing this risk entails when the intermediary operates as a merchant. Clearly, risk-averse intermediaries would have higher incentives to choose the referral mode.

That is, the action available to intermediary \( i \) in the first stage is to choose the set of buyers \( B_i \subseteq B \), to refer, and to potentially resell the object in the third stage to \( B_i \setminus B_j \).

For example, an analogous statement to Theorem 1 obtains, for example, if, in the trading phase there is no auction but the seller posts a take-it-or-leave-it price. As long as the seller sets the price after observing the business model chosen by the intermediaries, she will always ask at least an amount equal to the highest merchant-intermediary’s resale value; this intermediary will, in turn, prefer the referral mode.

Because intermediaries access different buyers, in order to achieve the integrated industry profit, a vertical agreement would require a complex scheme where intermediaries bid for the seller’s good after having sold the option to buy to buyers, or variants of this.

Recall that in an optimal second price auction, the reserve price is independent of the number of buyers and it thus the same reserve a merchant intermediary chooses.

However in a setup where the intermediary would find it optimal to use nonlinear tariffs, or where the seller would impose to the intermediary the use of such a tariff, Theorem 1, goes through as long as the upstream seller can choose an optimal reserve price in an second-price auction (SPA) or, more generally, has price flexibility and can price-discriminate across intermediaries.

To see this point note that, in view of part 2 of Theorem 2, if the intermediary sets \( \kappa = \frac{A(n, B)}{B} \), then each intermediary refers with probability 1. The cost for the seller is \( \kappa B = \frac{A(n, B)}{B} \) per intermediary, and this goes to zero as \( B \) gets large.

Note that \( t_i \) can be correlated with \( t_{-i} \) even if all prior information is independent, because \( t_i = (s_i, B_i) \) may include buyers referred by intermediary \( i \) who are among the participants in the mechanism and their types are in \( t_{-i} \).

As it is well-known from Myerson (1981), at the revenue-maximizing auction the buyer with the highest virtual valuation—rather than the buyer with highest valuation is awarded the good. The two coincide when all buyers valuations are identically distributed.

Theorems B.1 and B.2 were established for some fee regardless of how the fee is determined.

When intermediaries propose the referral fees, there also exist equilibria in which intermediaries do not always refer. For example, consider an equilibrium in which intermediary \( i \), regardless of his type, demands a referral fee above his expected per-buyer marginal value, that is, \( \frac{H - d_i}{\sum_j d_j} < \kappa_i^* \), and the seller, whenever he observes a referral fee different from \( \kappa_i^* \), believes that the intermediary has the highest possible resale value. In this equilibrium, the seller refuses every proposal, including \( \kappa_i^* \).


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