Vertical Mergers in Ecosystems with Consumer Hold-up*

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Abstract

An ecosystem comprises all downstream products that employ a certain upstream input. In many cases, final consumers make irreversible investments to join an ecosystem before downstream prices are set. By committing to buy products that use the specific ecosystem input, they are at risk of being held-up. Unable to observe future prices, consumers base their decisions on what they observe about the market structure within each ecosystem, including vertical contracts signed by the upstream firms. By entering into vertical agreements with multiple competing downstream firms, thus creating a credible expectation of lower prices, an upstream firm is able to mitigate consumers' hold-up problem and, as a result, increase ecosystem demand. Our main observation is that, in contrast to conventional wisdom, an upstream monopolist merging with one of its downstream affiliates will find it profitable to continue to serve downstream competitors, even when products sold downstream are homogeneous. In an appendix, we compute measures of upward pricing pressures following a merger in a market with such ecosystem effects.

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1 Introduction

A substantial body of academic work has emphasized that vertical mergers involving an upstream monopolist may result in complete *input* foreclosure of downstream firms, especially of those producing close substitutes of the integrated firm's final product (e.g., see Rey & Tirole (2007)).[1] The goal of this paper is to illustrate how this conclusion is overturned in the presence of consumer hold-up and ecosystem competition, or, more generally with "ecosystem effects".

We informally define an "ecosystem" as a supply-chain for the production of several final products. An ecosystem typically includes an upstream product (e.g., an operating system) and all downstream products that rely on it as an input or as an an essential complement with which they inter-operate (e.g., software applications). In our analysis, what distinguishes ecosystems from traditional supply chains is the potential for consumer *hold-up*.[2] That is, final clients decide, before prices of final products are set and for the long-term, which ecosystem to join. Being unable to observe future prices, consumers are at risk of being held-up and will rely on signals about their future consumer surplus, such as downstream market structure, in order to decide whether to patronize a certain ecosystem. Then, we speak of an upstream market featuring "ecosystem effects" when the *volume* of customers joining the ecosystem, i.e., committing to purchasing one of the ecosystem's downstream products (or otherwise incur switching costs) depends on *expected* consumer surplus, which in turn depends on the within-ecosystem *market structure* and, possibly, on the market structure of competing ecosystems.

For instance, consider the semiconductor industry. Chip-makers produce chips which are based on a specific architecture and sell them to manufacturers of final goods that need a CPU (e.g., laptop computers). Each architecture defines a separate ecosystem. Before purchasing chips manufacturers will have to invest in developing software and capabilities that will be only compatible with a specific architecture. This may occur before their purchases from chip-makers are finalized, for example because products are still under development. Because switching architectures requires sustaining start-up costs anew, the decision to join one or

[1] To the extent that the upstream monopolist is not able to frictionlessly extract the entire downstream surplus, the integrated firm benefits from foreclosure in two ways. First, foreclosure shifts profits away from competitors toward the integrated firm. Second, exclusion eliminates downstream competition, facilitating the extraction of downstream monopoly profits. Despite exclusion, integration often benefits consumers because, unless the downstream market is perfectly competitive or the upstream/downstream negotiation is frictionless, it eliminates double marginalization (see Tirole (1988)).

[2] An economic definition of ecosystem can be traced back to Moore (1993): "companies coevolve capabilities around a new innovation: they work cooperatively and competitively to support new products, satisfy customer needs, and eventually incorporate the next round of innovations." While the emphasis of Moore (1993) is on biological analogies, ours is on the strategic interlink of companies within the same supply-chains.

another architecture-based ecosystem commits them for the medium/long term and expose them to the risk of being held-up ex-post. Not knowing final chip-set prices, manufacturers will rely on signals, such as the number of chip-makers that have received a license to employ the specific processor architecture, among other considerations, to assess which ecosystem to join.

This papers focuses on the effects that a merger between an upstream monopolist who controls access to the ecosystem and one of its downstream affiliates (i.e., a downstream firm within the ecosystem) has on input supply and within-ecosystem competition. For example, the effects of a merger between the owner of an architecture and one of the chip-makers that produce chips based on such architecture.[3] Our main observation is the following. When ecosystem effects are present, the integrated upstream firm may find it beneficial to sign (non-foreclosing) contracts with downstream competitors, even with those selling close-substitutes. In fact, unless the integrated entity can sign long-term contracts with its final customers (which is often impractical as the downstream product might be still in development), it is unable to otherwise credibly promise low downstream prices in order to attract them into its ecosystem. Then, preserving within-ecosystem competition accomplishes the goal of credibly committing the integrated firm to keep within-ecosystem (downstream) prices low. This attracts consumers into the ecosystem, typically benefiting the upstream firm more than foreclosing downstream rivals.

These ideas may contribute to explain why vertical integration is often not followed by input foreclosure in industries that form business ecosystems, where customers at various levels of the supply chain sustain start-up costs, even in cases where standard theory would unambiguously predict otherwise (e.g., when downstream products enjoy high margins and are not sufficiently differentiated). For instance, when Google started producing mobile phones in competition with Apple, who was running a tightly closed ecosystem, it did not foreclose access to the Android operating system to competing phone producers. This strategic choice may have helped Google to grow the Android ecosystem, attracting consumers and software developers in a way that it would have not been possible had Google attempted to extract rents by monopolizing the downstream Android phone market with its own smart-phones.

We formalize our argument within the following model. An upstream firm moves first and makes take-it-or-leave-it offers to a set of downstream producers. At this stage, whether integrated or not, the upstream firm is, in principle, able to foreclose downstream producers by raising wholesale prices to a level where their production becomes unfeasible. Having observed the contracts between upstream and downstream producers, consumers decide whether to join the ecosystem or join an alternative one (i.e., exercise their outside option),

[3]As another example, consider an OS developer merging with an application developer, or a payment card scheme merging with an acquiring bank.

which for modeling's sake we assume is operated by a vertical integrated monopolistic firm.[4] Following these decisions, consumer valuations for the downstream product are realized and downstream competition takes place.

We stack the deck in favor of post-merger foreclosure by assuming that (a) downstream producers sell a homogeneous product and (b) that the interplay of upstream pricing and downstream competition does not fully dissipate downstream profits. Therefore, we treat the case where downstream profit can be extracted frictionlessly as a limit case (i.e., we allow the upstream firm to charge two-part tariffs, but not to extract the entire downstream profit through the fixed fee) and we assume competition takes place a' la Cournot, a scenario that results in positive profits for downstream firms.[5] First, regarding (a), foreclosure is rarely profitable if downstream producers sell differentiated products, which compete less intensely with the product of the integrated firm. Second, regarding (b), since in our model the upstream firm is already able to enforce a price level through wholesale pricing, the inability to fully extract downstream profit (e.g. either by means of perfect two-part tariffs or just because downstream firms compete intensely and make no profit) implies a motive for foreclosure, as downstream profit is what input foreclosing behavior is meant to appropriate.[6]

Within the model above, we obtain one general insight. When a market presents ecosystem effects, competing downstream firms will not be fully foreclosed post-merger. If any, input foreclosure will be partial.[7] Post-merger, the main economic trade-off for the upstream firm is one between *higher degree of downstream surplus appropriation*, which is achieved by raising the final price through higher wholesale prices, versus *higher volume*, which is achieved by means of a commitment to lower final prices, which in turn, requires serving competing firms (i.e., the lower price due to elimination of double marginalization is not sufficient). Under standard assumptions, we show that, at the margin, reducing the wholesale price below the fully-foreclosing wholesale price, which is equal to the downstream monopoly price, always benefits the integrated firm.

[4] Observability of vertical contracts is a crucial assumption but need not apply literally. In practice, it's important that consumers receive a signal of the competitiveness of the downstream market. For instance, which firms are affiliated to an ecosystem via long-term contracts (or licensing) and are developing products for its final market will usually be known to final consumers before they start making irreversible ecosystem-specific investments.

[5] With homogeneous products, Bertrand competition would leave no downstream profit, thus curbing incentive to foreclose.

[6] If the upstream firm lacks bargaining power, then the upstream firm might not able to enforce the desired price level and extract the entire downstream profit, *even* when downstream firms compete a' la Bertrand. There is a vast literature that models this scenario assuming the offers made by the upstream firm are secret. For example, see Hart et al. (1990), O'Brien & Shaffer (1992), McAfee & Schwartz (1994).

[7] Full foreclosure may happen in the limit if ecosystem differentiation (i) becomes so large that attempting to raise demand by lowering expected price is vain or (ii) it is so small that an epsilon extra expected surplus is sufficient to shift *all* possible demand away from competitors.

To obtain sharper results and a closed-form characterization, we focus on a case where both (i) the distribution of consumer valuations and (ii) the demand to join the ecosystem given expected prices are linear. We are then able to perform comprehensive comparative statics. Beyond the general result that there is no full foreclosure, we obtain the following additional results. First, we show that, as one would expect, wholesale prices are decreasing in the intensity of ecosystem effects (i.e., the degree of product homogeneity between the two competing ecosystems), and are increasing in the level of frictions in upstream/downstream pricing (i.e., the share of downstream profit that can be extracted via fixed fees). This implies an analogous negative relationship between intensity of ecosystem effects and within-ecosystem final price.

Second, we show that as the strength of ecosystem effects increases, the profit of downstream competitors converges to the pre-merger level, and so does the joint profit of the merging firms. Therefore, with strong ecosystem effects, we observe that the vertical merger may have little strategic impact on the profitability of the firms within the ecosystem. Welfare effects are shaped mainly by ecosystem effects, e.g., the intensity of between-ecosystem competition.[8] Intuitively, due to the importance of ecosystem effects, the merged firm will not have an interest in disadvantaging the non-merged downstream competitors, as this would severely affect the volume of consumers joining the ecosystem.

Finally, whether the merger is beneficial to the integrated firm, the downstream competitors and consumers, depends on the values of parameters. If, pre-merger, the ability to extract downstream surplus is below a certain middle-level threshold (e.g., at the extreme, if the upstream firm uses linear wholesale prices), then both the merged firm and consumers benefit from the merger. Following the merger, the upstream firm has more at stake in raising volume and therefore engages in more aggressive ecosystem competition for consumers. The incentives to grow the ecosystem are stronger and, in addition, double-marginalisation with one of the firms is eliminated. Downstream competitors also benefit from the merger if ecosystem effects are strong.

On the other hand, when a large share of downstream profit can be extracted without raising unit prices, via fixed fees, whether the merger is beneficial for consumers and the merging firms depends on the strength of ecosystem effects. Somewhat surprisingly, very strong ecosystem effects (i.e., intense ecosystem competition) may make the merger unprofitable and harm consumers. The intuition comes from the observation that the ability to impose non-linear prices implies the upstream firm has a good control of downstream behavior and sets very low unit wholesale prices, often below cost. But then, if this is the case, the

[8] Vertical integration is unlikely to be motivated by a desire to reestablish market power, when that power to raise prices is absent to begin with, due to strong inter-brand competition. With ecosystem effects due to hold-up problem of final consumers, strong inter-brand competition also results in unwillingness of the vertically integrated firm to extract intra-brand profits of downstream competitors.

merger may imply a binding loss of commitment to keeping prices low, as, effectively, the internal transfer-price must now raise to zero. This loss, which is greater when ecosystem effects are stronger, can be partially, but not fully, compensated post-merger by reducing even further the wholesale prices charged to the downstream competitors. In this case, while the merging firms do not jointly benefit from integration, nor normally do consumers, downstream competitors profit.

The paper contributes to the extensive literature on vertical control by highlighting a channel through which the provider of an essential input may *not* benefit from foreclosing downstream competition, even when doing so would allow to restore monopoly power in the downstream market. In particular, committing not to foreclose solves consumers hold-up, when the choice of ecosystem is ex-ante and there are switching costs. A somewhat related idea has been developed in Rey & Tirole (2007, Appendix A), although in their work it's downstream firms that suffer from a hold-problem. In particular, they sketch a model whereby downstream firms must sink investments that orient their technology toward one or some other upstream firm. They will then choose the upstream firm that guarantees them the highest profit. Because downstream firms face the risk of being foreclosed when dealing with an integrated upstream firm, the upstream firm may benefit from not integrating even if this results in its inability to enforce a monopoly outcome downstream. In a similar vein, but conversely, Chemla (2003) argues that downstream competition may protect the investment of the upstream firm against expropriation, when downstream firms have bargaining power. A crucial assumption in his model is that the upstream firm has a convex cost function, which implies that it makes a profit when bargaining with two or more firms having bargaining power and making take-it-or-leave-it offers, but not when it bargains with one.

The related idea that, with inter-brand competition, vertical separation can be used as a commitment device to *raise* prices has been explored in Bonanno & Vickers (1988), building on the seminal contribution on delegation by Vickers (1985). In their paper, upstream firms prefer to sell via single independent retailers. Separation confers the upstream firms with the ability to commit to raise downstream costs, which triggers a price increase that benefits every upstream firm when competition is in price, because prices are strategic complements (see Bulow et al. (1985)). When competition is in quantity, delegation is preferable to control from an individual-firm perspective, but in equilibrium pushes firms toward lower prices, because of strategic substitutability. However, in contrast to what happens in our model, suppressing intra-brand competition is not harmful to the ability of upstream firms to compete. In fact, as Rey & Stiglitz (1995) show, vertical restraints that establish a downstream monopoly, for instance exclusive territories or by excluding all but one of the downstream firms, can also be used in conjunction with commitment to soften inter-brand competition. We reach a different conclusion because competition is for-the-market and committing to low prices is only possible by affecting the downstream market structure.

We consider a setting where, by operating on consumer expectations, the presence of multiple downstream firms increases demand.[9] In practice, it is well documented that the possibility of second-sourcing makes products more attractive, mitigating both the risk of supplier-failure and hold-up. For instance, Li & Debo (2009) indicate that Apple normally chooses to acquire its inputs from at least two suppliers. Many other firms in the semiconductor industry have historically adopted a similar strategy (e.g., see Taylor (1984), Sirbu and Hughes (1986) and several issues of *Electronic News*). Explanations of second sourcing which are based on start-up costs and are closely related to ours have been previously advanced in the literature. In Shepard (1987) one monopolistic firm is able to contract on prices but not on quality. If consumers face start-up costs, then licensing its technology to a second firm is a credible commitment to provide high-quality and often raises total industry profit. In Farrell & Gallini (1988) a monopolist may find it in its interest to commit to open its technology, but with some delay, as a commitment to consumers who incur start-up costs. We share with these papers the general insight that committing to competition limits ex-post rent extraction and may increase demand. In contrast, our focus is on mergers in vertical structures and our set-up is much less stylized, because the signing of complex vertical contracts allows the integrated monopolist to, for instance, only partially foreclose downstream competitors.

Finally, related to our work is also the literature on "divisionalization", which studies motives for which firms may sell their products through multiple downstream divisions. Baye et al. (1996) Corchón (1991), Polasky (1992) consider the case of inter-brand competition for an homogeneous product, as we do in this paper, and study a two period game. First, two upstream firms choose the *number* of downstream retailers they want to have each. Second, all downstream firms compete a' la Cournot. In line with Bonanno & Vickers (1988), it is shown that, while they would jointly prefer a downstream duopoly, upstream firms have an individual incentive to increase the number of downstream firms to steal profit from their competitor, which ultimately results in a competitive equilibrium downstream. As in our model, the benefit from unilaterally dividing production among competing units, relies on upstream firms credibly committing to having those downstream units behave as independent profit-maximizers.

In the next section we present the model. In section 3 we conduct our general analysis, while section 4 focuses on the linear case (Appendix B presents the results of simulations for the linear case). Section 5 discusses a number of potential extensions and section 6 concludes the paper. Finally, in Appendix A we consider indexes of upward pricing pressures following

[9] Consumer expectations are a crucial determinant of demand also in the vast network effects literature. For instance, in Katz & Shapiro (1985) seminal paper a firm may want to commit to additional compatible downstream competition in order to encourage more consumers to join, which boosts network effects. Hence, ecosystem effects, as we have defined them, may arise from network effects (see Jorge Padilla (2021).

a vertical merger in a market with ecosystem effects (see Farrell & Shapiro (2010), Moresi & Salop (2013)). In particular, we show that upward pricing pressures indexes from vertical mergers should account for the mitigating effect of ecosystem effects. Raising downstream rivals costs has a negative effect on the volume of consumers and therefore reduces the incentives of the upstream firm to raise such prices post merger.

2 Model

A market is served by two ecosystems. The first ecosystem is open, with an upstream monopolist (U) serving identical downstream firms $1, \ldots, n$. The second ecosystem is closed and operated by a single integrated firm (I). Each ecosystem produces a homogeneous final product. However, the two ecosystems sell a different product.

The game we study develops in three periods. In the first period (*wholesale pricing stage*), U chooses two-part tariffs $(T_i, c_i)_{i=1}^n$ that it offers to all firms. Here T_i represents the fixed payment requested to i and c_i the wholesale price, or per-unit payment. For simplicity, we assume offers are observable and downstream firms must accept or reject them, simultaneously. Decisions are also observable and, if a firm rejects, it collects its reservation payoff of zero. The marginal cost of U is normalized to zero.

We introduce frictions in vertical negotiations by assuming that the upstream firm is constrained in the share of profit it can extract from downstream firms in the form of fixed fees. Let $\hat{\pi}_i(h)$ be the continuation equilibrium payoff of firm i following history h where offers have been made. We follow Calzolari et al. (2020) and assume offers will be rejected if $T_i > \lambda \hat{\pi}_i(h)$, where $\lambda \in [0, 1]$ is a parameter that measures frictions in upstream-downstream negotiations. In particular, $\lambda = 0$ corresponds to the case of pure linear wholesale prices, while $\lambda = 1$ corresponds to the traditional two-part tariffs scenario.[10]

In the second period (*between-ecosystem competition*), an (unbounded) mass of final buyers decide which ecosystem to join. At this stage, consumers are ex-ante identical, but, as we discuss later, they will learn about their heterogeneous individual values before making their downstream purchases.[11] A key assumption of the model is that the decision to join an

[10]This reduced-form approach can be provided with a solid foundation in several ways, discussed further in Calzolari et al. (2020). For instance suppose firms are risk neutral for both pure gains and losses, but their utility has a kink at zero, so that they are risk-averse when trading lotteries comprising losses and gains. Also, assume that with some probability downstream firms face no demand at all and otherwise everything is as normal. Then, the optimization problem of the upstream firm takes the form we adopt, with $1 - \lambda$ representing the degree of risk-aversion. As one would expect, if $\lambda = 1$ and downstream firms are risk-neutral, then all expected profits can be extracted by means of the fixed fee.

[11]Consumer heterogeneity generates a downstream downward sloping demand. The assumption that buyers make the decision to join the ecosystem without knowledge of their individual values avoids information-

ecosystem is irreversible.[12] Products are differentiated across ecosystems and, in order to join one or the other, consumers form expectations about the downstream surplus that they will obtain once product development by downstream firms is completed and competition among them takes place. Since in the downstream homogeneous-product market there's a one to one relationship between consumer surplus and final price, let's denote with $x(p_U^e, p_I^e) > 0$ the volume of consumers that choose U when they anticipate final prices to be p_U^e and p_I^e, for U and the integrated firm I respectively. We assume x is differentiable and strictly decreasing in the expected final price charged in the U ecosystem, $x' \equiv dx/dp_U^e < 0$.[13]

In the third period of the model (*within-ecosystem competition*) final prices are set in both ecosystems. Then, consumers learn about their values and purchasing decisions are made. In the U ecosystem, the final price p_U is determined by downstream competition between the n firms. We assume that downstream firms sell a homogeneous product and that outcomes are determined by the Cournot equilibrium condition. Being identical, all firms face, in addition to the wholesale price, the same marginal cost, which we assume constant and, without loss, normalize to zero. We assume that the value of each consumer is drawn from a continuous strictly increasing and log-concave CDF F in $[0, 1]$, admitting density f. This distribution also determines beliefs that firms have about consumers' valuations, when they make their pricing decisions.

We consider two variants of this game, one *pre-merger* scenario and one *post-merger*. The pre-merger scenario has been described above. In the post-merger scenario, U integrates with firm 1 and the model is as in the pre-merger case, except we maintain $c_1 = 0$ and $T_1 = 0$, which is common knowledge.[14] The assumption that the upstream firm cannot commit to a wholesale price for its subsidiary reflects the fact that integration partially reduces commitment and delegation power of the upstream firm.

We look at subgame perfect equilibria of this game and, to simplify the analysis, we focus on the case where all un-integrated downstream firms are offered the same contract. This is a restriction only if the objective of the upstream firm is not strictly concave pre or post-merger. Strict concavity is commonly assumed in applied work and it is satisfied, for instance, when demand is linear or, under the already stated assumptions, when $\lambda = 1$.

theoretic complications but is not essential for our main result to go through qualitatively.

[12]This assumption should not be taken literally. Switching costs need not be unbounded for our equilibrium analysis to continue to hold. Our results require the presence of substantial switching costs or the need for consumers to engage in meaningful ecosystem-specific investment.

[13]Being integrated, I takes x as given. Consumers make their decisions assuming that I charges zero to its downstream subsidiary, which operates as a monopoly. Because I maintains a passive role in the analysis, we don't need to impose further assumptions on x.

[14]As long as the decision to integrate is made ex-ante, endogeneizing the merger (without introducing additional frictions or benefits) simply implies evaluating whether the sum of payoffs of the upstream firm and the downstream firm that is integrated is larger pre or post-merger.

3 Analysis

We analyse this model by backward induction. Hence we start from within-ecosystem competition.

3.1 Within-ecosystem competition

Consider the last stage of the game and fix at $x \in (0, \infty)$ the volume of consumers that has chosen ecosystem U, depending on previous history.[15] At the within-ecosystem competition stage, firms sell homogeneous products and therefore a unique market price p will prevail. At that price, demand for the good produced by the U managed ecosystem will be equal to:

$$x[1 - F(p)].$$

Let q be the total non-negative quantity sold by all firms in the U ecosystem and let $q_{-i} = \sum_{j \neq i} q_j$.[16] Since F is strictly increasing, we can write the inverse demand function for $q \in [0, x]$ as

$$P(q) = F^{-1}(1 - \frac{q}{x}),$$

and, otherwise, let $P(q) = 0$ for $q > x$. Then, $(q_1, q_2, ..., q_n)$ is a *downstream Cournot equilibrium* if and only if for all $i = 1, \ldots, n$, it satisfies

$$q_i \in \arg\max_{y \in \Re^+} [P(y + q_{-i}) - c_i] y,$$

that is each firm is supplying the quantity of product that maximizes its profit, net of the already-sunk fixed fee T_i, taking as given the equilibrium supply of others firms.

It is well known that an equilibrium exists in this setting (see McManus (1964), Roberts & Sonnenschein (1976)) since log-concavity of F implies log-concavity of P and, as a consequence, strict concavity of firms' profit functions (see Bulow & Roberts (1989)). The equilibrium is also unique given log-concavity of demand and convexity of the cost functions (see von Mouche & Quartieri (2013)). Importantly, uniqueness implies that the continuation equilibrium following acceptance of offers is uniquely determined by the profile of marginal costs $(c_1, \ldots, c_2)$, with $c_i = \infty$ to indicate that the firm is not producing because it rejected an offer.

The next lemma shows that the equilibrium price is independent of the volume of consumers that has joined the ecosystem. While this is obvious in the monopoly case, where

[15] Volume x lies in the set $(0, \infty)$ because the function $x(\cdot, \cdot)$ is unbounded.

[16] Note that x determines the volume of consumers that can potentially buy (i.e., the maximum demand) while the actual quantity of consumers who purchase is determined by the equilibrium quantity decisions of downstream firms.

the monopolist maximizes $x[1-F(p)]p$ in p taking x as given, it deserves a short proof in the case of competing firms.

Lemma 1 *Fix $c_1, \dots, c_n$ and denote with $(q_1, \dots, q_n)$ an equilibrium for $x = 1$. Then, $(xq_1, \dots, xq_n)$ is an equilibrium for $x > 0$ and $P(x(q_1 + \cdots + q_n)) = F^{-1}(1 - (q_1 + \cdots + q_n))$ is the equilibrium price, which does not depend on x.*

Proof The second part of the statement is obvious once we have proved the first. A necessary condition for equilibrium for firm i is

$$\frac{q_i}{x} \frac{1}{f\left(F^{-1}(1 - \frac{q_i + q_{-i}}{x})\right)} = F^{-1}(1 - \frac{q_i + q_{-i}}{x}) - c_i.$$

If this holds for $(q_1, \dots, q_n)$ for $x = 1$, it will also holds for $(xq_1, \dots, xq_n)$ when $x \neq 1$. $\square$

This lemma simplifies the analysis considerably. It implies that, for both ecosystems, consumers expectations of prices are not determined by the volume of consumers that join one ecosystem or another, but by market structure and the contracts that have been signed by the downstream firms.[17]

Before proceeding, we present another lemma that turns out to be useful. While it is immediate to see that an increase in the marginal cost of a firm reduces that firm's production and increases production of other firms, we now show that under the assumed log-concavity, an increase in the cost of any individual firm also reduces *total* quantity produced.

Lemma 2 *Total quantity produced decreases when the marginal cost of any firm producing a strictly positive quantity increases.*

Proof A proof for the case of a common cost shift for symmetric firms is in Seade (1985), but it is straightforward to extend it to the case of individual shifts for heterogeneous firms (see also Dixit (1986)). Let's return to the first order conditions from Lemma 1, for $i = 1, \dots, n$ we have

$$\frac{q_i}{xf\left(F^{-1}(1 - \frac{q_i + q_{-i}}{x})\right)} = F^{-1}(1 - \frac{q_i + q_{-i}}{x}) - c_i.$$

Summing up the FOCs for all firms, assuming they all produce a non-zero quantity, the following must hold

$$nF^{-1}(1 - \frac{q}{x}) - \sum_i c_i - \frac{q}{xf\left(F^{-1}(1 - \frac{q}{x})\right)} = 0,$$

[17] This property differentiates further our setup by one with classic network effects. With network effects, the value that consumers expect to obtain from the ecosystem would depend on the volume, thus providing stronger incentives to lower prices.

where we write total quantity as $q = \sum_i q_i$. Using $P(q) = F^{-1}(1 - q/x)$ we get

$$Z(q, c) = nP(q) - \sum_i c_i + qP'(q) = 0.$$

We can then use the implicit function theorem to obtain

$$\frac{\partial q}{\partial c_i} = -\frac{\frac{\partial Z}{\partial c_i}}{\frac{\partial Z}{\partial q}} = \frac{1}{(n+1)P'(q) + qP''(q)},$$

which is negative because log-concavity of F implies $2P'(q) + qP''(q) < 0$ and a fortiori $(n+1)P'(q) + qP''(q) < 0$ since $P' < 0$. □

3.2 Between-ecosystem competition

We can then proceed backward to the between-ecosystem competition stage, at which consumers make their ecosystem decision choices. Since, I operates as an integrated firm, we maintain that the prevailing price downstream will be the monopoly price p^M, that solves $p^M = (1 - F(p^M))/f(p^M)$.[18] On the other hand, the price in U will be determined by the number of active downstream firms and pricing choices of the upstream firm.

For $x = 1$, for any profiles of costs $(c_1, \ldots, c_n)$, we denote the unique downstream equilibrium as $(\hat{q}_1(c_1, \ldots, c_n), \ldots, \hat{q}_n(c_1, \ldots, c_n))$ and we maintain the mapping is continuous as it is, for instance, in the linear case. Let the equilibrium price be $\hat{p}(c_1, \ldots, c_2)$ and recall $\hat{\pi}_i(c_1, \ldots, c_n) = \hat{q}_i(c_1, \ldots, c_n) \left(\hat{p}(c_1, \ldots, c_n) - c_i\right)$ is the equilibrium profit of downstream firm i. Quantity and profit of downstream firm i at $x \neq 1$ will simply be $x\hat{q}_1(c_1, \ldots, c_n)$ and $x\hat{\pi}_i(c_1, \ldots, c_n)$.

Since the downstream prices for both ecosystems are, in any continuation equilibrium, equal to $\hat{p}(c_1, \ldots, c_n)$ and p^M, consumer preferences imply that the total volume for the U ecosystem is equal to

$$\hat{x}(c_1, \ldots, c_2) = x(\hat{p}(c_1, \ldots, c_n), p^M).$$

3.3 Wholesale-pricing stage

We are now in a position to characterize the optimal upstream decision, which takes place at the wholesale pricing stage.

[18] We assume that I operates with the same upstream cost as U. This assumption is not crucial. Introducing such cost asymmetries would not affect our main results.

Our first observation is that we can focus attention to equilibria where all offers are accepted and this simply requires the upstream firm to set $T_i \leq \lambda \hat{x}(c_1, \ldots, c_n)\hat{\pi}_i(c_1, \ldots, c_n)$. To see this, note that downstream firms always strictly prefer to accept offers such that $T_i < \lambda \hat{x}(c_1, \ldots, c_n)\hat{\pi}_i(c_1, \ldots, c_n)$ and are indifferent when this holds with equality. Hence, when an upstream firm would have its offer rejected by firm i, there is another payoff-equivalent equilibrium where it sets $c_i = \infty$ and $T_i = 0$ and that offer is accepted.

In light of the above, in the *pre-merger* scenario, U solves

$$\max_{\{T_i, c_i\}_{i=1,\ldots,n}} \hat{x}(c_1, \ldots, c_n) \sum_i c_i \hat{q}_i(c_1, \ldots, c_2) + \sum_i T_i \tag{O.Pre}$$

subject to:

$$T_i \leq \lambda \hat{x}(c_1, \ldots, c_n)\hat{\pi}_i(c_1, \ldots, c_n) \text{ for } i = 1, \ldots, n$$

In the *post-merger* scenario U internalises the profit of the integrated downstream firm 1 and maximises

$$\max_{\{T_i, c_i\}_{i=2,\ldots,n}} \hat{x}(0, c_2, \ldots, c_n) \left[\hat{\pi}_1(0, c_2, \ldots, c_n) + \sum_{i=2}^{n} c_i \hat{q}_i(0, c_2, \ldots, c_n)\right] + \sum_{i=2}^{n} T_i \tag{O.Post}$$

subject to:

$$T_i \leq \lambda \hat{x}(0, c_2, \ldots, c_n)\hat{\pi}_i(0, c_2, \ldots, c_n) \text{ for } i = 2, \ldots, n$$

We remark that c_i should be interpreted as a markup over upstream marginal cost, rather than as absolute cost value. Negative values then imply that the upstream firm sells below marginal cost, not necessarily that it is making a payment to downstream firms.

The following lemma is immediate and, therefore, stated without proof. It says that it is without loss of generality to assume that all participation constraints are binding.

Lemma 3 *In a pre-merger equilibrium* $T_i = \lambda \hat{x}(c_1, \ldots, c_n)\hat{\pi}_i(c_1, \ldots, c_n)$ *for* $i = 1, \ldots, n$ *and in a post-merger* $T_i = \lambda \hat{x}(0, c_2, \ldots, c_n)\hat{\pi}_i(0, c_2, \ldots, c_n)$ *for* $i = 2, \ldots, n$.

3.4 Main Result

In this section we present our main result, Proposition 1. To begin with, suppose there are *no* ecosystem effects and x is independent of the costs charged to downstream firms. For instance $x = 1$, because there's no between-ecosystem competition and the volume of consumers is independent of expected prices. It is easy to see that in this case, for any $\lambda < 1$, the solution to (O.Pre) involves all firms being treated symmetrically and operating in equilibrium.[19] On

[19]If $\lambda = 1$ all firms will make zero-profit and the upstream firm essentially operates as an integrated firm, so, without ecosystem effects, it might as well shut-down some of the non-integrated downstream firms.

the other hand, it is well known that the solution to (O.Post) involves no production and zero profit for all non-integrated firms when there are *no ecosystem effects*. There is *complete input foreclosure* post-merger. Moreover, it can be shown that post-merger the integrated firm implements the monopoly outcome. We emphasise this result in the following Lemma.

Lemma 4 *Let $x(p_U^e, p_I^e) = 1$ and $\lambda < 1$. Then (O.Post) is solved by $T_i = 0$, $c_i \geq p^M$ for $i > 1$. At a solution $c_2^*, \ldots, c_n^*$, we have $\hat{q}_i(0, c_2^*, \ldots, c_n^*) = 0$ for $i \neq 1$ and $\hat{p}(0, c_2^*, \ldots, c_n^*) = p^M$.*

A couple of clarifications are in order. First, in light of Lemma 1, the result extends to the case where $x \neq 1$ but is independent of prices. Second, when $\lambda = 1$, even post-merger, the upstream firm is indifferent between any combination of $(0, c_2, \ldots, c_n)$ that delivers $\hat{p}(0, c_2, \ldots, c_n) = p^M$. Hence, while possible, foreclosure is not needed to maximize profits of the integrated firm when downstream profit can be fully extracted by means of two-part tariffs.

This result is conventional wisdom and we do not provide a complete proof. However, it is instructive to look at the argument in some details for the case of two firms, 1 and 2.

Proof Assuming $x(p_U^e, p_I^e) = 1$ and noting that the participation constraint of 2 will hold with equality at the optimum, (O.Post) becomes

$$\max_{c_2} \hat{\pi}_1(0, c_2) + c_2 \hat{q}_2(0, c_2) + \lambda \hat{\pi}_2(0, c).$$

Recall that $\hat{\pi}_i(c_1, c_2) = \max_y [P(y + \hat{q}_{-i}(c_1, c_2)) - c_i]y$. Differentiating with respect to c_2, the *envelope theorem* implies $\hat{\pi}_1' = P'\, \hat{q}_2'\, \hat{q}_1$ and $\hat{\pi}_2' = P'\, \hat{q}_1'\, \hat{q}_2 - \hat{q}_2$, where, of course, the derivative of P is over quantity and the derivative of quantities are over c_2. Economizing on notation, the first order condition for the maximization problem above can then be expressed as

$$\hat{\pi}_1' + c_2 \hat{q}_2' + \hat{q}_2 + \lambda \hat{\pi}_2' = 0 \;\; \textit{or}$$

$$P'\, \hat{q}_2'\, \hat{q}_1 + c_2 \hat{q}_2' + \hat{q}_2 + \lambda P'\, \hat{q}_1'\, \hat{q}_2 - \lambda \hat{q}_2 = 0.$$

Cournot optimality conditions for both firms require $P'\hat{q}_1 = -P(\hat{q}_1 + \hat{q}_2) = P'\hat{q}_2 - c_2$. Recalling that $\hat{p} = P(\hat{q}_1 + \hat{q}_2)$, and introducing further notation $\tilde{\pi}_U'(c_2)$ for the first-derivative of the profit function of U absent ecosystem effects, we can then rewrite the above first-order condition as

$$\tilde{\pi}_U'(c_2) = -(\hat{p}(0, c_2) - c_2)(\hat{q}_2'(0, c_2) + \lambda \hat{q}_1'(0, c_2)) + (1 - \lambda)\hat{q}_2(0, c_2) = 0. \tag{1}$$

Then observe that $\tilde{\pi}_U'(c_2) \geq 0$ because it follows from Lemma 2 that $\hat{q}_2' + \lambda \hat{q}_1' < 0$, since $\lambda < 1$. Hence, raising the cost of the rival downstream firm always benefits the integrated firm U, up until the competing downstream firms all produce zero.

Since $\tilde{p} \geq c_2$ implies $\hat{q}_2 = 0$, the solution to (O.Post) has $c_2 \geq \hat{p}(0, c_2) = p^M$. Firm 2 makes no profit following integration while the integrated firm accrues monopoly profit since $P(\hat{q}_1(0, p^M)) = p^M$. Without ecosystem effects, foreclosure following integration is total. □

Our goal is to contrast the result above with the case in which ecosystem effects are at play, that is, consumers patronize ecosystems based on their expectations of downstream prices, which are driven by the observed market structure in the U ecosystem. Our main result is that, under the stated assumptions, *complete input foreclosure* will not take place with ecosystem effects. At the margin, starting from the full-foreclosure outcome, the upstream firm will have an incentive to lower the cost of one or more downstream competitors. This has the effect of lowering the final price, thus reducing the margin for the merged firm, and, for any fixed volume, it may also lower the share of total output sold by the merged firm. However, when between-ecosystem competition is taken into account, the increase in volume, which is driven away from the integrated ecosystem I, benefits the merged firm. Without loss, we focus on a symmetric equilibrium, where non-integrated firms all receive the same offer.

Proposition 1 *At the symmetric optimum of (O.Post) the upstream firms chooses wholesale price c such that $c < \hat{p}(0, c, \ldots, c) < p^M$.*

Before proceeding with the proof, we highlight that the volume of consumers that joins the ecosystem is a decreasing function of the wholesale price charged to downstream firms.

Lemma 5 *For any c such that $\hat{p}(0, c, \ldots, c) \geq c$ (i.e., as long as non-integrated downstream firms do not have a strict incentive to shut down production), we must have $\frac{d\hat{x}}{dc}(0, c, \ldots, c) < 0$, while otherwise $\frac{d\hat{x}}{dc}(0, c, \ldots, c) = 0$.*

Proof Observe that $\frac{d\hat{x}}{dc} = \hat{x}' = x'\hat{p}'$ where $x' < 0$ is the derivative of x with respect to $\hat{p}$. Note that $\hat{p}' = (\hat{q}_1{}' + \hat{q}_2{}')P' > 0$ since $P' < 0$ and $\hat{q}_1{}' + \hat{q}_2{}' < 0$ by lemma 2, as long as non-integrated firms produce. If non-integrated downstream firms are not producing, then marginally changing c_2 has no effect on the quantity produced nor on the expected price. □

The above follows because raising downstream rivals' costs decreases overall quantity following Lemma 2 and therefore raises the expected price perceived by consumers, which reduces the number of them that joins the ecosystem. We can now return to the proof of proposition 1.

Proof of Proposition 1 We develop the argument for two firms, but it readily extends to the n firms case. Consider (O.Post). Substituting for the binding participation constraint and collecting $\hat{x}$ the objective function becomes

$$\hat{x}(0, c_2)[\hat{\pi}_1(0, c_2) + c_2\hat{q}_2(0, c_2) + \lambda\hat{\pi}_2(0, c_2)]$$

Recalling the analysis performed in the $x = 1$ case, differentiate the objective function with respect to c_2 to obtain the following first-order condition

$$\tilde{\pi}'_U = -\frac{\hat{x}'}{\hat{x}}\tilde{\pi}_U, \tag{2}$$

where $\tilde{\pi}_U$ is the profit of the integrated upstream firm when $\hat{x} = 1$, $\tilde{\pi}'_U$ is defined in equation 1 and the derivative of $\hat{x}$ is with respect to c_2.

Recall from Lemma 5 that $\hat{x}' < 0$ in the open region of costs up to zero production of downstream firms. Then observe $\hat{x} > 0$ and $\hat{\pi}_U > 0$. Hence, right-hand side of 2 is non-negative.

Now, consider π'_U in 1 and observe that $\hat{p}(0,0) > 0$ and that $\hat{p}(0, c_2)$ is increasing in c_2 as we argued in the previous paragraph. Hence $\pi'_U > 0$ except when $\hat{p}(0, c_2) \geq c_2$ when we have $\pi'_U = 0$. Since $\pi'_U(p^M) = 0$ and for lower prices it is positive, we conclude that an optimum $c_2^* < \hat{p} < p^M$. Foreclosure is not full. □

There's a simple mathematical intuition for this result. For fixed volume, the profit of the upstream firm increases in c (i.e., the wholesale price to non-merged downstream firms) but it reaches a plateau when $c = p^M$, that is with complete foreclosure. Then, of course, raising c further does not increase the profit of the upstream firm. However, while the effect of lowering c from the full-exclusion level is negligible on the ability to extract profit for a given level of volume when ecosystem effects are muted (since $\pi'_U(p^M) = 0$), the marginal increase in profit that comes from additional volume stolen from the competing integrated ecosystem, that is $\frac{\hat{x}'}{\hat{x}}\hat{\pi}_U$, is non-negligible when consumers have valuations that come from a log-concave distribution. Hence, the upstream firm always benefits from setting c in such a way that downstream competitors produce a positive quantity. Of course, as we highlight in the next section where we consider the linear case, how low is the optimal c depends on how weak are the ecosystem effects and how easy is to extract downstream profits (i.e., λ).

We remark that non-foreclosure of the downstream competitors occurs because the only way the upstream firm is able to commit to low-prices, and therefore attract consumers to its ecosystem, who choose before downstream prices are finalized, away from the competing ecosystem, is by committing to contracts with downstream firms. Instead, suppose consumers could observe final prices *before* committing to one ecosystem or the other. In this case, we would have full foreclosure of downstream competitors, as the integrated upstream firm would be able to keep prices low in order to harness ecosystem effects without giving shares to competitors.

4 The linear case

Having presented our main result, in this section we consider a simple linear setting with two firms, 1 and 2, to obtain closed form solutions and engage in comparative statics.

First, in order to model within-ecosystem competition we assume consumers decide between ecosystems following a simple linear demand structure with differentiated products (e.g. see Levitan & Shubik (1972)), where the volume for the ecosystem U is given by

$$x(p_U^e, p_I^e) = 1 - (p_U^e - p_I^e)/\beta$$

with $\beta > 0$. It follows that $x' = -1/\beta < 0$. The lower β, the more the two ecosystems compete with each other, the larger are ecosystem effects.[20]

Second, we assume that the willingness to pay of individual consumers is uniformly distributed, with $F(v) = v$ for $v \in [0, 1]$. Under the uniform distribution, we have $p_I^e = p^M = 1/2$. Observe that the uniform distribution is log-concave and induces strictly concave profit functions and a unique equilibrium, since the associated reaction functions intersect only once.

Equilibrium quantities and prices for $x = 1$ take the following classic form:

$$\hat{q}_1(c_1, c_2) = \frac{1 + c_2 - 2c_1}{3}; \quad \hat{q}_2(c_1, c_2) = \frac{1 + c_1 - 2c_2}{3}; \quad \hat{p}(c_1, c_2) = \frac{1 + c_1 + c_2}{3},$$

and observe that $\frac{\partial q_1(c_1,c_2)}{\partial c_2} + \frac{\partial q_2(c_1,c_2)}{\partial c_2} = -\frac{1}{3}$ for $c_2 \in [0, (1 - c_1)/2)$, as predicted by lemma 2.

After substituting for the binding participation constraints of firms 1 and 2 and for the expected prices, and noting that the solution to concave (O.Pre) will be symmetric, we can rewrite linear (O.Pre) and (O.Post) as follows

$$\max_c \left[1 + \left(\frac{1}{2} - \frac{1 + 2c}{3}\right)/\beta\right]\left[2\frac{1 - c}{3}c + 2\lambda\left(\frac{1 - c}{3}\right)^2\right] \tag{ℓ O.Pre}$$

$$\max_{c_2} \left[1 + \left(\frac{1}{2} - \frac{1 + c_2}{3}\right)/\beta\right]\left[\frac{1 - 2c_2}{3}c_2 + \frac{1}{9}(1 + c_2)^2 + \frac{\lambda}{9}(1 - 2c_2)^2\right] \tag{ℓ O.Post}$$

The linear case satisfies our condition for concavity of the upstream profit functions, which establishes that the solution will indeed be symmetric. It is a matter of computation to show that the optimal symmetric c in the pre-merger case is

$$\hat{c}(\beta, \lambda) = \frac{\beta}{2} + \frac{\sqrt{\lambda^2 + 4(\lambda - 3)^2\beta^2 - 4(\lambda - 3)(\lambda - 1)\beta - 2\lambda + 13} + 3\lambda - 5}{4(\lambda - 3)}$$

[20] An analogous demand would also arise from an Hotelling-type model of product differentiation and price competition, e.g. see Mathewson & Winter (1984)

while c_2 in the post-merger case is

$$\hat{c}_2(\beta, \lambda) = \frac{1}{2} + \beta - \sqrt{\beta^2 + \frac{3}{4(5 - 4\lambda)}}.$$

As predicted by Proposition 1, we have the following result that foreclosure post-merger will not be complete.

Remark 1 $\hat{c}_2(\beta, \lambda) < 1/2 = p^M$.

Also, note that $\lim_{\beta \to \infty} \hat{c}_2 = 1/2 = p^M$. That is, as positive ecosystem effects disappear, i.e., there is no between-ecosystem competition and consumers are locked into the ecosystem, we are back to the classic foreclosure scenario.

These values can then be used to obtain a closed form solution to our key variables, including price and quantities. Focusing on the linear case, we can highlight the following results, which can all be easily verified with some algebra or simulations (see Appendix B).

Remark 2 *Wholesale prices increase when the intensity of competition between ecosystems (i.e., strength of ecosystem effect) is reduced (i.e., β increases), both pre and post-merger. That is $\frac{\partial \hat{c}}{\partial \beta} > 0$ and $\frac{\partial \hat{c}_2}{\partial \beta} > 0$.*

In contrast to the classic setting without ecosystem effects, when λ is sufficiently high, we may observe wholesale prices going below marginal cost of the upstream firm at lower levels of β (i.e., strong ecosystem effects), as the upstream firm attempts to harnesses ecosystem effects by inducing a low downstream price and intensifying competition with the rival ecosystem. The remark below complements the above.

Remark 3 *Wholesale prices decrease when the share of downstream profits that can be appropriated via fixed fees, λ, increases. That is $\frac{\partial \hat{c}}{\partial \lambda} < 0$ and $\frac{\partial \hat{c}_2}{\partial \lambda} < 0$.*

This is a standard result, which continues to hold with within-ecosystem competition and ecosystem effects. The easier is for the upstream firm to extract profit via the fixed fee, the more it can reduce the distortions created by positive wholesale price. The next result is more surprising.

Remark 4 *As the strength of ecosystem effects grows, the profit of the downstream competitor converges to the pre-merger scenario* $\lim_{\beta \to 0} \mid \hat{\pi}_2(\hat{c}, \hat{c}) - \hat{\pi}_2(0, \hat{c}_2) \mid = 0$

This result suggests that there might be minimal to no foreclosure following a vertical merger in an industry exhibiting strong ecosystem effects.

Remark 5 *As the strength of ecosystem effects grows, the profit of the integrated firm converges to the pre-merger scenario*

$$\lim_{\beta \to 0} \mid 2\left(\lambda \hat{\pi}_1(\hat{c}, \hat{c}) + \hat{c}\hat{q}_1(\hat{c}, \hat{c})\right) - \left(\hat{\pi}_1(0, \hat{c}_2) + \lambda \hat{\pi}_2(0, \hat{c}_2) + \hat{c}_2 \hat{q}_2(0, \hat{c}_2)\right) \mid = 0$$

This result suggests that with strong ecosystem effects there is little gain from strategic monopolization. Combining the last two remarks, we see that with strong ecosystem effects vertical integration has minor (strategic) effects on firms profitability, which is driven by the need to keep prices at a level that maximizes the ecosystem's value.

The next result focuses on market outcomes following integration, which are in general ambiguous and depend on the interplay of the exogenous parameters.

Remark 6 *For any $\lambda \in [0, 1]$ there exists non-decreasing $\frac{4\sqrt{2}-3}{2} \geq \tau_p(\lambda) \geq 0$ such that for all $\beta > \tau_p(\lambda)$, the post-merger final price is* below *the pre-merger price (equivalently $\hat{c}_2 \leq 2\hat{c}$), and consumers benefit from the merger as a result. Otherwise, for $\beta < \tau_p(\lambda)$, the price increases post-merger and consumers are worse off.*[21]

Note that $\tau_p(\lambda) = 0$ for $\lambda \leq 1/2$. Hence, when downstream surplus extraction by means of two part-tariffs is sufficiently imperfect, the merger always benefits consumers. It turns out that the merging firms are also better off, while the downstream competitor is worse off. This observation and a description of welfare outcome for the case $\lambda \geq 1/2$ is contained in the next remark.

Remark 7 *a) For $\lambda \leq 1/2$, the merger makes consumers and the merging firms better off, while the downstream competitor worse off.*

b) For $\lambda > 1/2$, there exists thresholds $\tau_p(\lambda) > \tau_U(\lambda) > \tau_2(\lambda)$ such that

(b1) for $\beta > \tau_p(\lambda)$ post-merger price is below pre-merger while for $\beta > \tau_p(\lambda)$ it is above;

(b2) for $\beta > \tau_U(\lambda)$ the merger is profitable and for $\beta < \tau_U(\lambda)$ the merger is unprofitable;

(b3) for $\beta > \tau_2(\lambda)$ firm 2 is worse off and for $\beta < \tau_2(\lambda)$ is better off than pre-merger;

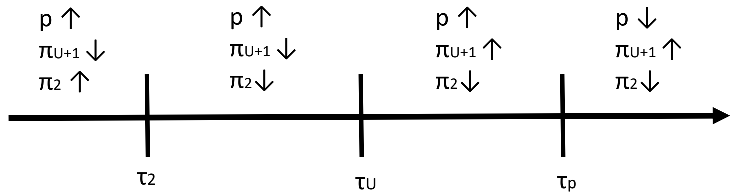
So, for $\lambda < 1/2$ or $\lambda > 1/2$ and $\beta > \tau_p(\lambda)$ the merger benefits consumers and merging firms, while the downstream competitor is worse off. For $\beta < \tau_2(\lambda)$, the merger makes consumers and the merging firm worse off, while making the downstream competitor better off. [22]

To simplify the interpretation of this proposition, the case of $\lambda > 1/2$ is summarized in the figure below. Note that $\tau_p(\lambda)$ is below 1.33, so it is somewhat small in relative terms.

[21] $\tau_p(\lambda) = 0$ for $\lambda \leq 1/2$ and for $\lambda \geq 1/2$ we have $\tau_p(\lambda) = \frac{2\sqrt{\lambda^2+2\lambda-1}}{5-4\lambda} - \frac{2\lambda+1}{2(5-4\lambda)}$

[22] Remarkably, for $\lambda > 1/2$ and $\beta \in (\tau_2, \tau_U)$ all economic agents are worse off from the merger. The merger is profitable while consumers are worse off only when $\beta \in (\tau_U, \tau_p)$.

Figure 1: Effects of the merger on price, profit of the merged firms, π_{U+1}, and downstream competitor, as a function of β for $\lambda > 1/2$.



The intuition for this result is as follows. When $\lambda < 1/2$, the inability to extract revenues through fixed fees implies that the upstream firm needs to keep wholesale prices high enough to earn profits. Then, pre-merger, wholesale prices are always above-cost. In this case, integration, which pushes to zero the internal wholesale price for the integrated firm, can only increase the ability of the upstream firm to steer volume to the ecosystem post-merger. Because post-merger the upstream firm internalizes more of the total profit from driving consumers to the ecosystem, integration provides incentives to boost ecosystem effects and to lower price further. As a result of this, and of the fact that double marginalization within the integrated firm is eliminated, consumers benefit. The upstream firm also benefits because it can always replicate the pre-merger price while making a higher profit, by raising the wholesale price and steering quantity away from the competitor. This explain why the competitor is worse off from the merger.

Instead, consider the case where the upstream firm can extract most of the downstream profit through a fixed fee (i.e. $\lambda > 1/2$). In this case, we need to distinguish between strong and weak ecosystem effects, or in other words, high or low level of between-ecossytem competition intensity. With weak ecosystem effects, wholesale prices are above cost pre-merger and the effects are the same as those outlined in the previous paragraph. Instead, when ecosystem effects are strong, the upstream firm has, pre-merger, an incentive to bring wholesale prices below cost, in order to stimulate ecosystem effects. Now, when integration takes place, the integrated firm loses its ability to commit to giving below-cost prices to the merged downstream firm. Hence, ceteris paribus, integration results in an increase in price. Then, in order to harness ecosystem effects again and steal volume from the competing ecosystem, the upstream firm has post-merger an incentive to lower wholesale price for the downstream competitor further than pre-merger levels. However, since because of classic business stealing the upstream firm now gains from reducing quantity of downstream competitor firm, the price may not return down to the pre-merger level (i.e., when $\beta < \tau_p$). Hence, consumer may end up being worse off. When this happens, the integrated firm tends to be also worse off from the merger (i.e., when $\beta < \tau_U$), due to the binding loss of commitment power. For the reasons outlined above, instead, the downstream competitor tends to benefit from the merger (i.e., when $\beta < \tau_2$).

The table below summarises the effects of integration to the various parties (integrated $U+1$ and 2), depending on the strength of both intensity of within-ecosystem competition (i.e., ecosystem effects) and upstream/downstream frictions.

Table 1: Effects of Vertical Integration: pre-merger vs post-merger payoff

	Weak Effects ($\beta > \tau_p$)	**Strong Effects** ($\beta < \tau_2$)
Low frictions ($\lambda > 1/2$)	Consumers $\uparrow$, $U + D_1 \uparrow$, $D_2 \uparrow$	Consumers $\downarrow$, $U + D_1 \downarrow$, $D_2 \uparrow$
High frictions ($\lambda < 1/2$)	Consumers $\uparrow$, $U + D_1 \uparrow$, $D_2 \downarrow$	Consumers $\uparrow$, $U + D_1 \uparrow$, $D_2 \downarrow$

For completeness, we present graphically in Appendix B the results of simulating the main equilibrium variables where we vary the strength of ecosystem effects for three scenarios: (1) $\lambda = 0$ pure linear pricing, (2) $\lambda = 1/2$ imperfect two part tariffs and (3) $\lambda = 1$ unrestricted two-part tariffs. To serve as reference, we remind that in the linear case the monopoly consumer surplus 0.125 and the monopoly price is 0.5.

5 Extensions

Bertrand within-ecosystem competition. Suppose downstream firms compete in prices. Before integration, the upstream firm can control the downstream price by setting a common wholesale price equal to the price it desires. Moreover, it will extract all the downstream profit. While it would be able to implement the monopoly outcome, by the same argument we made for Cournot competition, the upstream firm will want to lower wholesale prices below the monopoly level, to steal demand away from the integrated ecosystem. Let's now suppose the upstream firm merges with one of the downstream firms. Nothing changes from the perspective of the integrated firm, as it can still enforce the desired price by charging wholesale prices to the downstream firms. In short, as it is known, when the downstream market is fully competitive the upstream monopolist can implement its desired outcome and extract all profit, both before and after the merger. As such the merger has no effect on consumers or the downstream firms alike. This case is equivalent to the one where unrestricted two-part tariffs can be demanded by the upstream firm by means of take-it-or-leave-it offers.

Differentiated within-ecosystem products. We have assumed that the downstream firms sell a homogeneous product. While often unrealistic for typical ecosystems, this assumption provides the best-shot at exclusion taking place. The analysis in this paper would go through, a fortiori, if downstream products were differentiated. To accommodate this case, the model could be extended by assuming that consumers have a randomly distributed taste for variety and choose one ecosystem or the other based on the expected average price they face downstream.[23] The existence of multiple products produced by different firms would unambiguously reduce incentives to foreclose post-merger. In fact, compared to the homogeneous product case, foreclosure now shrinks the surplus that can be extracted.

Lack of contract observability. We motivated the existence of ecosystem effects by assuming that consumers opt into an ecosystem after having observed some signal about the contracts signed by downstream firms. Moreover, once they join an ecosystem, they become captive, at least in the medium term. While it is often the case that upstream firms engage in actions to publicize their vertical contracts (e.g., Apple and Google often do), the assumption of observability of contracts cannot simply be disposed of without affecting equilibrium. If consumers cannot observe contracts at all, then they must join an ecosystem or the other independently of the upstream/downstream contractual choices. In this scenario, however, the upstream firm will fully foreclose downstream competitors. Anticipating this, consumers will expect both ecosystems to price monopolistically. This implies that both ecosystems will do so, and integration will be followed by exclusion. Hence, we conclude, complete lack of observability prevents the upstream firm from harnessing ecosystem effects.[24]

6 Conclusions

This paper argues that competition between upstream firms that control competing ecosystems may take place by them shaping the downstream within-ecosystem industry structure.

In many ecosystems, customers incur elevated switching costs when moving from one ecosystem to another, and often make irreversible ecosystem-specific investments ahead of the realization of final prices which are relevant to them. This creates a hold-up problem, once membership in one ecosystem has been taken up. When the firm running an ecosystem is unable to commit to final prices, which may happen for a variety of reasons, it will want to use upstream contracts to signal to consumers that they will not be held up. Hence, an upstream firm that controls an ecosystem may find it useful, as a way of competing more effectively with other ecosystems, to commit to serving several downstream firms operating in competition with each other. This will attract consumers, providing them with some confidence that the final price will be kept low by competition.

One conclusion that can be drawn from our analysis is that input-foreclosure concerns may be overstated for vertical mergers in industries characterized by ecosystem effects, for

[23] For instance, assume the individual consumer has valuations for two products distributed according to joint CDF $F(v_1, v_2)$. Then, within ecosystem demand function for product $i = 1, 2$ would be computed as $D_i(p_1, p_2) = \Pr\{v_i - p_i > 0, v_i - p_i > v_{-i} - p_{-i}\}$.

[24] This is equivalent to the effect of lack of observability of the investment in classic hold-up situations (e.g., see Gul (2001) and Condorelli & Szentes (2020)).

instance when there is between-ecosystem competition and consumers risk being held-up once they make their ecosystem membership decisions. Our findings bear relevance for the assessment of mergers in a number of high-tech industries, personal computer and smartphone industries, as well as other business ecosystems.

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Appendix A: Measures of Upward Pricing Pressure in Vertical Mergers with Ecosystem effects

In this appendix we compute measures of upward pricing pressure following a vertical merger in a market with ecosystem effects as defined in this paper. We follow the general approach developed in Farrell & Shapiro (2010). Suppose two firms merge but continue to operate independently, except the common owner will be able to impose a per-unit tax to each of them. In the case of an upstream firm, that would mean a tax on units of input sold to non-merged downstream firms. Pricing pressure indicators measure how large would that tax have to be to convince the independently managed firms to make the jointly optimal decision, starting from pre-merger variables. Noting that a per-unit tax raises marginal costs of firms and therefore final prices, Farrell & Shapiro (2010) argue that this represent a valid measure of the incentives that these firms would have, post-merger, to raise prices by distorting their choices away from the pre-merger equilibrium. We refer to them for a full motivation of this methodology.

In this appendix, we apply logic above to define upward pricing pressure indexes in vertical mergers. The analysis we conduct follows Moresi & Salop (2013) and adapts their framework, built on Farrell & Shapiro (2010), to the case of ecosystem effects. We restrict attention to the case of a *monopolist* input supplier and two downstream firms.[25] In addition to incorporating ecosystem effects, we extend their analysis by considering imperfect but observable two-part tariffs, following Calzolari et al. (2020).

Differentiated products As the case of differentiated products is perhaps the more relevant one from an empirical perspective, we begin by considering a version of our model with differentiated products, which we outlined in Section 5. To this end, let's denote with $x(p_1^e, p_2^e)$ the volume of consumers joining the ecosystem, where p_i^e represents expected price of within-ecosystem firm i and where prices from the competing ecosystem are taken as given. Naturally, we assume x is decreasing in both expected prices. Then, we let $D_i(p_1, p_2)$ be the within-ecosystem demand, per unit-consumer, going to firm i, given realized within-ecosystem prices p_1 and p_2 (see footnote 23 for a derivation of this demand system).

In our model, an input supplier makes take-it-or-leave-it offers of two-part tariffs to two downstream manufacturers, who then choose prices for their final good. Let's call c_i the wholesale price charged to firm i, and assume fixed fees are set to extract the maximum profit possible (see Lemma 3 for details). Let λ be the share of downstream profit that can be extracted by means of fixed fees from downstream firms. For added usability of our indexes, we set the upstream marginal cost to c_U, despite we normalized it to zero in the main text. We maintain the downstream marginal production costs to zero.

Next, assume the upstream firm merges with firm 1. As a default, $c_1 = c_U$ post-merger as the upstream firm loses the power to commit to a different internal price. The post-merger problem for the merged firm is sequential. Starting backward, from the consumer pricing period, the integrated firm best-responds by maximizing

$$x(p_1^e, p_2^e)\left[D_1(p_1, p_2)(p_1 - c_U) + D_2(p_1, p_2)(c_2 - c_U)\right]. \tag{3}$$

in p_1 taking as given the price of firm 2. Note that in choosing p_1 the integrated firm also takes the ecosystem effect as given because consumers have made their choices already. We denote this by writing x as a function of expected prices. Moreover, fixed fees do not enter the maximization problem of 1 as, we assume, they have already been paid by firm 2.

In the first period, the integrated firm chooses c_2 to maximize

$$x(p_1, p_2)\left[D_1(p_1, p_2)(p_1 - c_U) + D_2(p_1, p_2)(\lambda p_2 + (1 - \lambda)c_2 - c_U)\right], \tag{4}$$

subject to downstream equilibrium pricing functions $p_1(c_1, c_2)$ and $p_2(c_1, c_2)$, with derivatives over c_2 henceforth denoted $p_i'(c_2)$ for $i = 1, 2$.

[25] We refer to Moresi & Salop (2013), and in particular at their analytical appendix, for the more general analysis.

Differentiating with respect to p_1 the value function 3 and setting to zero gives the following condition

$$x(p_1^e, p_2^e)\left[\frac{\partial D_1(p_1,p_2)}{\partial p_1}(p_1 - c_U) + D_1(p_1,p_2) + \frac{\partial D_2(p_1,p_2)}{\partial p_1}(c_2 - c_U)\right] = 0. \tag{5}$$

Before proceeding with obtaining first-order condition for the upstream firm, we introduce the following notation

$$x'(c_2) = \frac{\partial x(p_1(c_2), p_2(c_2))}{\partial p_1}\frac{dp_1(c_2)}{dc_2} + \frac{\partial x(p_1(c_2), p_2(c_2))}{\partial p_2}\frac{dp_2(c_2)}{dc_2}$$

$$D_i'(c_2) = \frac{\partial D_i(p_1(c_2), p_2(c_2))}{\partial p_1}\frac{dp_1(c_2)}{dc_2} + \frac{\partial D_i(p_1(c_2), p_2(c_2))}{\partial p_2}\frac{dp_2(c_2)}{dc_2}.$$

Here, $x'(c_2)$ represents the marginal effect of changing c_2 on the volume of consumers. Normally, $x'(c_2) < 0$ as a raise in cost of firm 2 will tend to increase both prices, assuming prices are strategic complements. D_i' stands for the marginal effect of raising c_2 on the demand of the product of firm i. Note that, normally, $D_1' > 0$ and $D_2' < 0$ because, despite both prices may increase, a raise in the cost of product 2 will tend to reduce the demand of product 2 and raise demand of product 1.

Then, differentiating 4 with respect to c_2 gives the following *post-merger* first order condition

$$\begin{aligned} x(p_1,p_2)[D_1'(c_2)(p_1 - c_U) + D_1(p_1,p_2)p_1'(c_2) + \\ D_2'(c_2)(\lambda p_2 + (1-\lambda)c_2 - c_U) + D_2(p_1,p_2)(\lambda p_2'(c_2) + (1-\lambda))] + \\ x'(c_2)\left[D_1(p_1,p_2)(p_1 - c_U) + D_2(p_1,p_2)(\lambda p_2 + (1-\lambda)c_2 - c_U)\right] = 0. \end{aligned} \tag{6}$$

Next, we look at a post-merger scenario, but assume firms behave independently subject to centrally controlled taxation. In particular, per-unit taxes t_U, is imposed on the upstream firm per-unit on the production allowed for firm 2, and t_D, is imposed on the own production of firm 1. Starting backward again, the objective functions of the two firms 1 and U are respectively

$$x(p_1^e, p_2^e)D_1(p_1,p_2)(p_1 - c_1 - t_D), \tag{7}$$

$$x(p_1,p_2)[D_1(p_1,p_2)(\lambda p_1 + (1-\lambda)c_1 - c_U) + D_2(p_1,p_2)(\lambda p_2 + (1-\lambda)c_2 - c_U - t_U)], \tag{8}$$

where note that firm 1 is offered the good at price c_1 because firms are behaving independently even in respect of internal pricing.

Next, we proceed with obtaining first-order conditions for the two firms. Consider first the downstream firm 1. We have

$$x(p_1^e, p_2^e)\left[\frac{\partial D_1(p_1,p_2)}{\partial p_1}(p_1 - c_1 - t_D) + D_1(p_1,p_2)\right] = 0. \tag{9}$$

In the case of the upstream firm, differentiating by c_2 and setting to zero we get

$$\begin{aligned} x(p_1,p_2)[D_1'(c_2)(\lambda p_1 + (1-\lambda)c_1 - c_U) + D_1(p_1,p_2)\lambda p_1'(c_2) + \\ D_2'(c_2)(\lambda p_2 + (1-\lambda)c_2 - c_U - t_U) + D_2(p_1,p_2)(\lambda p_2'(c_2) + (1-\lambda))] + \\ x'(c_2)\left[D_1(p_1,p_2)(\lambda p_1 + (1-\lambda)c_1 - c_U) + D_2(p_1,p_2)(\lambda p_2 + (1-\lambda)c_2 - c_U - t_U)\right] = 0, \end{aligned} \tag{10}$$

where we assume $p_1'(c_2)$ is the same as the one arising from 5 because the upstream firm anticipates that the tax on the downstream firm will make it behave as if they were integrated.

To obtain our upward pricing pressure index (UPPI) measures in the presence of ecosystem effects, let's first define the relevant standard *diversion ratio*, $DR_{\{2,1\}}$ and the *ecosystem diversion ratio*, $eDR_{\{1,2\}}$. We have

$$DR_{2,1} = -\frac{\frac{\partial D_2(p_1,p_2)}{\partial p_1}}{\frac{\partial D_1(p_1,p_2)}{\partial p_1}}.$$

This is called *diversion ratio* because it measures the increase in the demand of firm 2 following an increase in price of firm 1, measured as a fraction of the demand that leaves 1 following the price increase. Normally, $eDR_{2,1} > 0$ because increasing the price of product 1 reduces the demand of product 1 and increases that of product 2. the See Farrel and Shapiro (2010) and Moresi and Salop (2013) for more on the diversion ratio in a setting where altering the cost c_2 has no effect on the pricing of firm 1.

Then, for the ecosystem case, we have

$$eDR_{1,2} = -\frac{x(p_1,p_2)D_1'(c_2) + x'(c_2)D_1(p_1,p_2)}{x(p_1,p_2)D_2'(c_2) + x'(c_2)D_2(p_1,p_2)}$$

The ratio $eDR_{1,2}$ measures how much of the shift in volume generated by an increase in c_2 that is lost by firm 2 is going to accrue to firm 1, including ecosystem effect. Compared to the case with no ecosystem effects, where $x = 1$ and $x'(c_2) = 0$, the diversion need not be positive. In fact, the denominator is always negative, but the numerator might not be positive if the ecosystem effects, as measured by negative $x'(c_2)$ are very strong. In general, the diversion ratio is now mitigated by the effect that raising the price of the other firm has on the ecosystem, which is negative and proportional to the initial demand.

We can now define our UPPI measure that account for ecosystem effects. First, $evUPP_U$, the ecosystem vertical UPP for the upstream firm, is obtained by solving for t_U by equating

(7) and (11), while $evUPP_d$, for the downstream firm, is obtained by equating (5) and (9) and solving for t_U. Recalling $c_1 = c_U$ when the upstream firm is considered, but pre-merger $c_1 \neq c_U$ for the downstream firm, we get

$$evUPP_U = t_U^* = eDR_{1,2}(1-\lambda)(p_1 - c_1) - (1-\lambda)p_1'(c_2)\frac{x(p_1,p_2)D_1(p1,p_2)}{x(p_1,p_2)D_1'(c_2) + x'(c_2)D_1(p_1,p_2)},$$

$$evUPP_d = t_d^* = -(c_1 - c_U) + DR_{2,1}(c_2 - c_U).$$

Now observe the following. First, ecosystems effects are not relevant for the definition of the $evUPP_d$. This is because at the time downstream firms chooses, ecosystem volume x is taken as given. Second, the smaller is $\lambda \in [0,1]$ the higher is the upward pricing pressure for U. If λ is large, the upstream firm already internalizes the profit of the non-merged downstream firm before the merger, so that there is less pressure to raise the price post-merger. Third, the most relevant change brought by ecosystem effects in the definition of upward pricing pressure indexes for the upstream firm consists on the fact that ecosystem effects affect the relevant diversion ratio. Fifth, the sign of the second term in the $evUPP_U$ depends on whether prices are strategic substitutes or complements, that is on whether $p_1'(c_2)$ is negative or positive (note: this assumes $dp_2/d_c2 > 0$). Since the fraction is always negative, strategic complementarity increases the pricing pressure.[26] Finally, $(c_1 - c_U)$ represents the size of elimination of double marginalization in the case of pricing for the downstream firm and mitigates the pricing-pressure as it represents already an important differential advantage for the own product.

These measure can be then appropriately normalized to obtain gross measures as follows

$$evGUPPI_U = eDR_{1,2}(1-\lambda)\frac{p_1 - c_1}{c_2} - \frac{D_1(p1,p_2)p_1'(c_2)}{c_2[x(p_1,p_2)D_1'(c_2) + x'(c_2)D_1(p_1,p_2)]},$$

$$evGUPPI_d = -\frac{c_1 - c_U}{p_1} + DR_{2,1}\frac{c_2 - c_U}{p_1}.$$

Finally, we can obtain a measure of price pressure on the rival downstream firm. That is

$$evUPPI_r = evUPPI_U \times PTR_U$$

where PTR_U represents the cost pass-through for the upstream firm and therefore transforms the extra taxation cost for the upstream firm into an actual price increase. Normalizing by p_2 one obtains the $evGUPPI_r$.

[26] This term does not appear in Moresi and Salop (2013), which consider the pricing of firm 1 unaffected by the contract signed by firm 1, even post-merger.

Homogeneous Products We next analyze the homogeneous product Cournot case which is our baseline model. In this case, since products are homogeneous, the ecosystem volume multiplier depends on the unique downstream price, that is we write $x(P(q_1^e, q_2^e))$, and the superscript e still stands for expected values.

The post-merger profit for the upstream firm is

$$x(P(q_1+q_2))\left[(P(q_1+q_2)-c_U)q_1+(\lambda P(q_1+q_2)+(1-\lambda)c_2-c_U)q_2\right],$$

where q_1 and q_2 are determined in the second stage and are function of c_1 and c_2. The post-merger profit function of the integrated downstream firm is

$$x(P^e)\left[(P(q_1+q_2)-c_U)q_1+(c_2-c_U)q_2\right]$$

where $x(P^e)$ depends on previous-period expected prices and is therefore considered fixed.

Let

$$P'(c_2)=\frac{\partial P}{\partial q_1}\frac{\partial q_1}{\partial c_2}+\frac{\partial P}{\partial q_2}\frac{\partial q_2}{\partial c_2}=\frac{dP}{dq}\left(\frac{\partial q_1}{\partial c_2}+\frac{\partial q_2}{\partial c_2}\right)$$

represent the marginal variation in price following a change in c_2 and note this is negative for log-concave demand (see Lemma x). Also writing $P=P(q_1+q_2)$, the first order conditions with respect to c_2 for the upstream firm is

$$x(P)\left[\frac{dq_1}{dc_2}(P-c_U)+P'(c_2)q_1+(\lambda P+(1-\lambda)c_2-c_U)\frac{dq_2}{dc_2}+q_2(\lambda P'(c_2)+(1-\lambda))\right]$$
$$+\frac{\partial x}{\partial P}P'(c_2)\left[(P-c_U)q_1+(\lambda P+(1-\lambda)c_2-c_U)q_2\right]=0 \quad (11)$$

The best-response for the downstream firm instead is

$$x(P)\left[\frac{\partial P}{\partial q_1}q_1+P-c_U\right]=0$$

where x is taken as given and so is q_2. This gives us the Cournot condition $-\frac{\partial P}{\partial q_1}q_1=P-c_U$.

We next write down the problem of individual firms subject to fictional taxation. Let's start with the objective functions of the upstream firms subject to a tax t_U on quantity offered by the competing firm 2. This is

$$x(P)\left[(\lambda P+(1-\lambda)c_1-c_U)q_1+(\lambda P+(1-\lambda)c_2-c_U-t_U)q_2\right].$$

By taking the first-order condition with respect to c_2 we obtain

$$x(P)\left[\frac{dq_1}{dc_2}(\lambda P+(1-\lambda)c_1-c_U)+\lambda P'(c_2)q_1+\frac{dq_2}{dc_2}(\lambda P+(1-\lambda)c_2-c_U-t_U)+q_2(\lambda P'(c_2)+(1-\lambda))\right]$$
$$+\frac{dx}{dP}P'(c_2)\left[(\lambda P+(1-\lambda)c_1-c_U)q_1+(\lambda P+(1-\lambda)c_2-c_U-t_U)q_2\right]=0. \quad (12)$$

where again the downstream firm is assumed to behave as the integrated one would, due to the optimal level of taxation. Equating (11) and (12) and simplifying we obtain

$$x(P)\left[(1-\lambda)(P-c_1)\frac{dq_1}{dc_2}+(1-\lambda)P'(c_2)q_1+\frac{dq_2}{dc_2}t_U\right]+\frac{\partial x}{\partial P}P'(c_2)\left[(1-\lambda)(P-c_1)q_1+t_Uq_2\right]=0. \tag{13}$$

$$\left[x(P)\frac{dq_2}{dc_2}+\frac{\partial x}{\partial P}P'q_2\right]t_U=-x(P)(1-\lambda)\left[(P-c_1)\frac{dq_1}{dc_2}+P'(c_2)q_1\right]-\frac{\partial x}{\partial P}(1-\lambda)P'(c_2)(P-c_1)q_1.$$

Then, define the ecosystem diversion ratio in the Cournot model as

$$eDR^c_{1,2}=-\frac{x(P)\frac{dq_1}{dc_2}+\frac{\partial x}{\partial P}P'(c_2)q_1}{x(P)\frac{dq_2}{dc_2}+\frac{\partial x}{\partial P}P'(c_2)q_2}$$

and solve for t_U to get

$$evUPP_U(Cournot)=(1-\lambda)(P-c_1)eDR^c_{1,2}-(1-\lambda)\frac{x(P)P'(c_2)q_1}{x(P)\frac{dq_2}{dc_2}+\frac{\partial x}{\partial P}P'q_2}$$

Note that, as expected, the fraction is equal to the classic diversion ratio without ecosystem effects, because in that case $\frac{\partial x}{\partial P}=0$.

Now consider the downstream merged firms, subject to a tax t_d on quantity produced. The objective function is

$$x(P^e)[(P(q_1+q_2)-c_1-t_d)q_1].$$

and we can obtain the following first-order condition

$$x\left[\frac{\partial P}{\partial q_1}q_1+(P-c_1-t_d)\right]=0. \tag{14}$$

Equating (6) and (14) and solving for t_d we obtain

$$evUPPI_d(Cournot)=t^*_D=-(c_1-c_U)<0.$$

As one would expect, with linear prices the downstream firm has no incentive to change quantity in a Cournot setting, because it does not help the upstream firm since it anticipates the quantity of other downstream firms will not react. As for the differentiated product case, ecosystem effects have no bite. Dividing by P one obtains $evGUPPI_d(Cournot)$.

Finally, we can obtain a measure of price pressure on the rival downstream firm. That is

$$evUPPI_r=evUPPI_U\times PTR_U$$

where PTR_U is the pass-through of the upstream firm. Normalizing by P one obtains the $evGUPPI_r$.

Appendix B: Simulations

Figure 2: $\lambda = 0$ — Outcomes as function of β, Pre-merger in blue and Post-merger in orange.

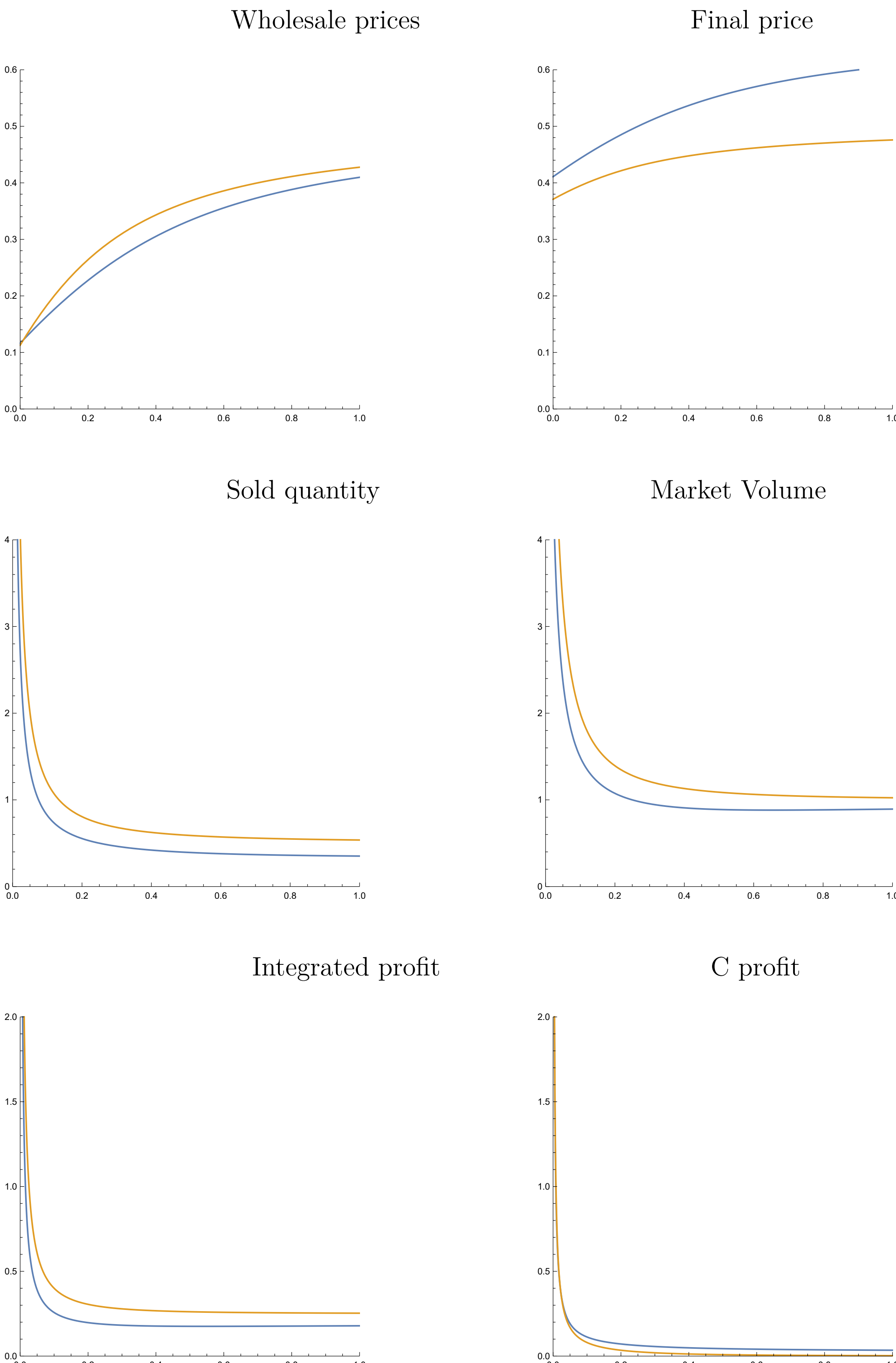
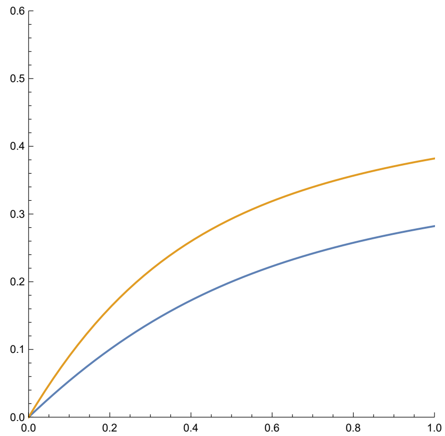
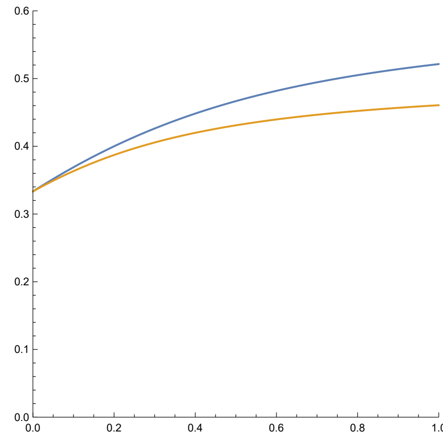


Figure 3: $\lambda = 1/2$ — Outcomes as function t, Pre merger in blue and Post-merger in orange.

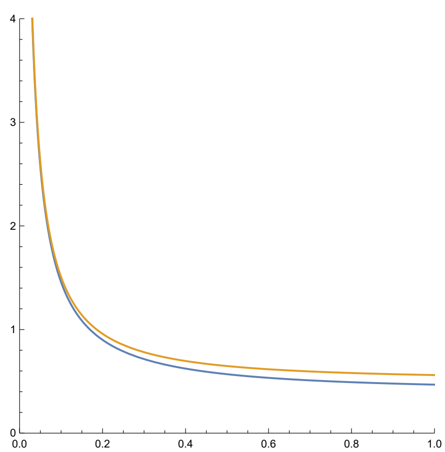
Wholesale prices



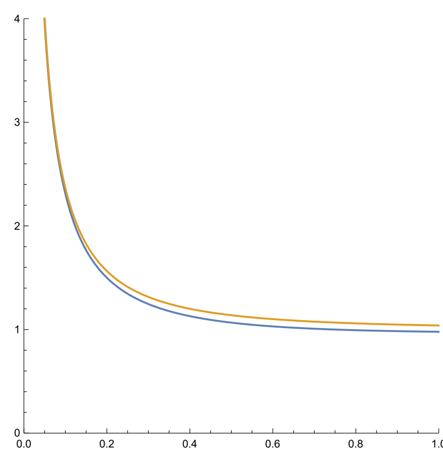
Final price



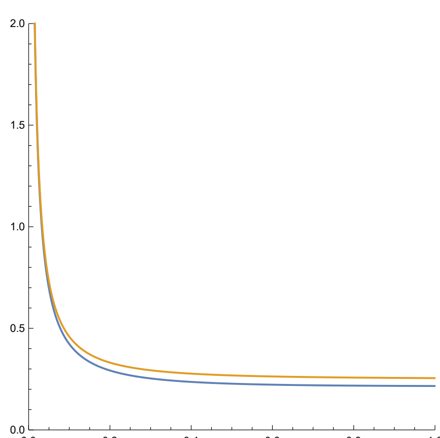
Sold quantity



Market Volume



Integrated profit



C profit

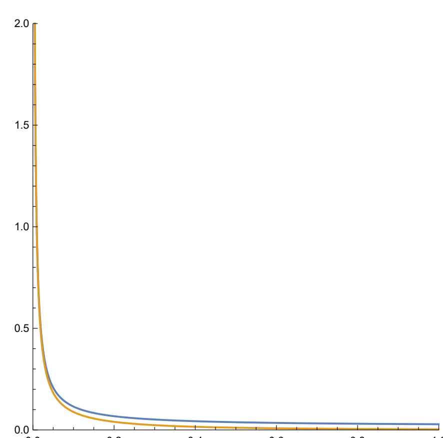


Figure 4: $\lambda = 3/4$ — Outcomes as function of β, Pre-merger in blue and Post-merger in orange.

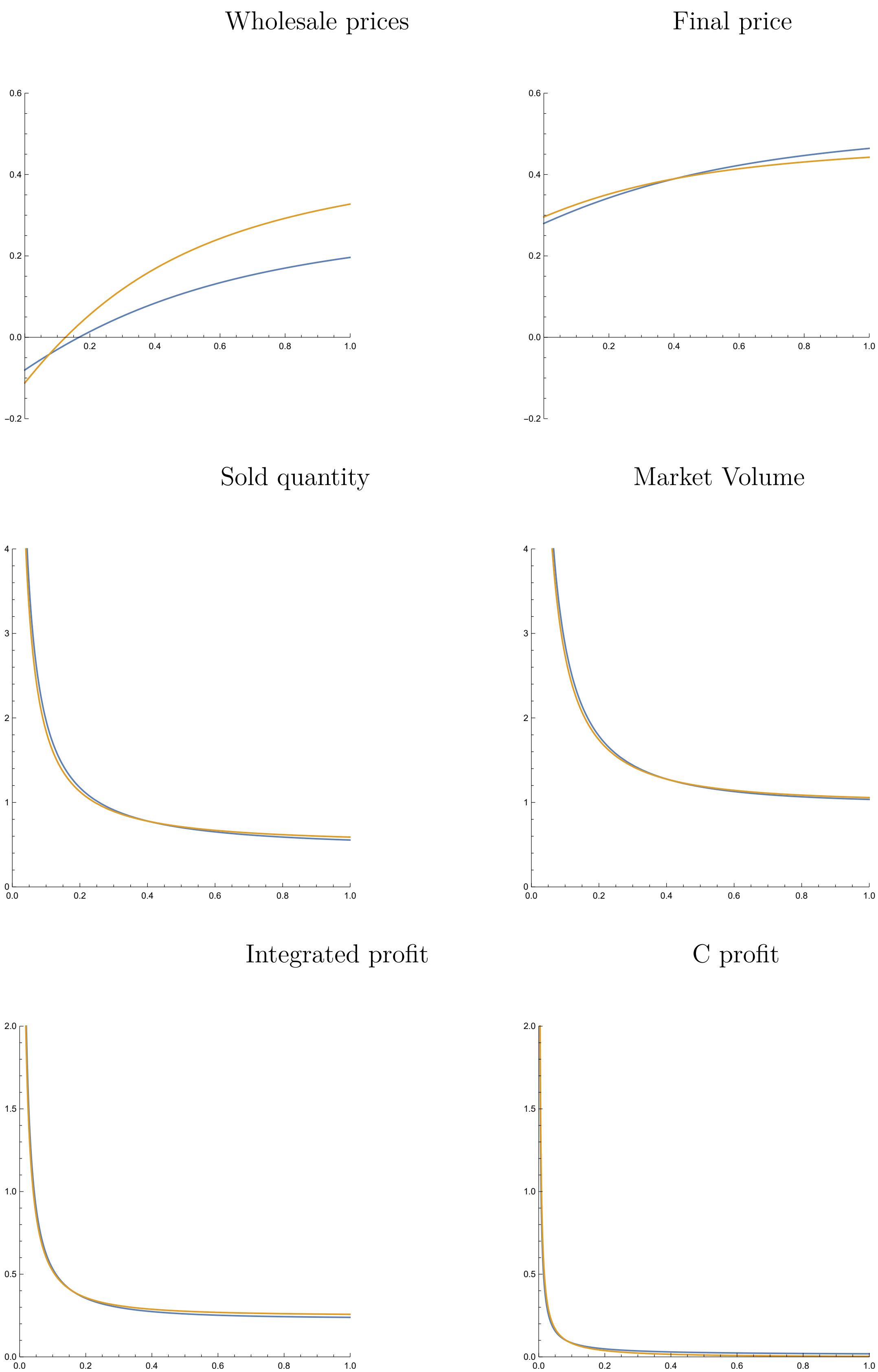


Figure 5: $\lambda = 1$ — Outcomes as function of β, Pre-merger in blue and Post-merger in orange.